Small and disruptive: Non-Sticky Insured Deposits and Banking System Stability. [*]

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Abstract

Recent empirical works by Martin et al. (2018) document that banks facing failure compensate for departing depositors by attracting insured deposits. This run-in reduces the bank's liability cost and enhances its chances of weathering distress. However, it also weakens the discipline imposed by depositors and leads banks to take more risks. This paper introduces a theoretical model assessing the impact of insured deposit flows on the overarching stability of the banking system. The model underscores that sophisticated insured depositors seeking to maximize returns amplify the gravity of a banking crisis. Remarkably, even a minimal fraction of these non-sticky depositors can introduce destabilizing effects. In this context, I argue that regulatory measures limiting insured deposit flows, like strict controls on brokered deposits, are an all-or-nothing game. Unless they drastically reduce the activity of insured depositors, they are ineffective in sustaining the discipline deposits impose on bank risk-taking.

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1 Introduction

The 2023 banking crisis underscored the critical role of the lethargic behavior of depositors in maintaining banks' stability (see Koont et al. (2023)). In the background of the crisis lies a drastic interest rate surge of approximately 500 basis points between February 2020 and May 2023. This hike eroded the value of bank loans and other debt instruments issued in the low-interest-rate environment that, barring a brief deviation between 2017 and 2019, had prevailed since 2008. The saving grace of many banks was the fact that the rates they had to pay for their deposit accounts did not reflect the new interest rate landscape. Instead, depositors demonstrated inertia: They clung to their banks and accepted near-zero rates, despite the opportunity to earn higher returns elsewhere. This was even true for insured depositors, who faced little risk-related consequences of switching banks. However, a subset of more sophisticated players actively pursued higher returns on their deposits. The behavior of such sophisticated depositors placed additional pressure on banks that relied heavily on their funds, including Silicon Valley Bank and First Republic Bank, which failed. The role of these depositors in the crisis generated growing concerns about new technologies that reduce the cost associated with transitioning deposits between banks. Specifically, it raised concerns for the safety of banks with advanced digital technology that allows an easier immediate withdrawal of deposit funds.

In this paper, I argue that the risk imposed on the banking system by active depositors does not lie only in the risk of a run-off on specific banks. Rather, it lies in such players' use of deposit insurance in run-ins that provide a distressed bank with cheap funding. The role of insured deposit run-ins in replacing other funds that runoff from a bank at risk of failure was recently documented by Martin et al. (2018). As they mention, this mechanism weakens the discipline that depositors impose on banks and worsens moral hazard. Here, I explore this channel of impact in more detail and argue that it greatly exacerbates the severity of tail events. Using a theoretical model, I show that the "safety net" insured players provide to distressed banks collapses during a system-wide crisis. In such times it does not contribute to the ability of banks at risk to raise liabilities at a lower price and wither through their distress. However, the protection it gives banks from exposure to idiosyncratic shocks in normal times leads them to enter the crisis with a more risky portfolio. That, in turn, leads to more bank failures.

The model demonstrates a few other facts about the impact of mobile insured deposits on stability. First, it demonstrates that even a very small group of more savvy insured depositors is sufficient to substantially undermine the banking system's stability. Second, it shows that the collapse of the safety net provided by such depositors is more dramatic than the failure of other hedging mechanisms, such as insurance, when facing a large aggregate shock. Insurance may fail to *fully* deliver its promises when overburdened. In contrast, due to its decentralized nature, the hedging provided by insured depositors means that in a crisis is *does not provide any benefits at all*. Third, I use the model to demonstrate that a feedback loop between the rise in risk and a rise in the liabilities can lead to a bank collapse that seems otherwise solvent and liquid. That is, it generates an empirical pattern that resembles a bank run, although it originates from a different source than the coordination failure mechanism often discussed in the literature (see **?**). In this context, I argue that models that suggest that runs are the result of multiple equilibria generated by the interdependence between the interest rate on liabilities and the risk of bank failure, such as Egan et al. (2017), hinge upon the usage of an imperfect equilibrium framework that is underpinned by somewhat problematic assumptions about depositors' behavior.

In the model, banks source funds from both insured and uninsured depositors. They strategically pick the risk level of their investment portfolio. As the portfolio's performance gradually unfolds, uninsured depositors require a risk premium for banks that face a substantial risk of default. Conversely, most insured depositors exhibit inertia, remaining loyal to their bank regardless of financial health. Yet, a minor proportion of insured depositors proactively pursue banks that proffer the highest interest rates, indifferent to the bank's default probability. In equilibrium, distressed banks endeavor to substitute high-risk premium uninsured deposits with funds accrued from these proactive insured depositors. This behavior in equilibrium aligns with the empirical findings of Martin et al. (2018), who observed uninsured deposit outflows being counterbalanced by insured deposit inflows in financially distressed banks.

In this setting, bank runs, that is, deposit outflows leading to a bank failure, stem from a feedback loop between the bank's risk of failure and the interest rates. An elevated likelihood of bank failure incites a rise in interest rates to appease depositors, thereby increasing the probability of failure due to the augmented liability burden. This mechanism bears significant similarity to the failure mechanisms explored in the sovereign default literature through the lens of the Eaton-Gersovitz (1981) model (Eaton and Gersovitz (1981)).

I utilize the model to explore the impact of active insured depositors on bank failure. Such depositors, who are protected from the risk of losing their funds if the bank defaults, do not require higher returns when a bank exhibits signs of weakness. Thus, they provide banks with cheaper funding in periods of distress, improving their prospects of withering through them. By that, they lower failure rates. However, the cheaper funding also means that banks incur a lower penalty, in the shape of higher interest on their liabilities, for exposing themselves to risk. The lower penalty exacerbates the moral hazard problem between banks and depositors by leading banks to hold riskier portfolios. By that, they render banks more likely to fail. Typically, these two opposing forces balance each other out, and the influx of insured deposits into troubled banks has an uncertain impact on failure rates. I show that the result of their joint operation is ambiguous. However, the moral hazard factor tends to dominate when the marginal return on bank investment diminishes at a low rate. For small banks with but a negligible impact on the marginal return of investment opportunities in the market, it seems likely to assume this is the case.

However, the more interesting phenomenon occurs during a crisis. In such times, distressed banks' demand for deposits dramatically increases. These banks outbid each other in competing for the pool of cheaper funds offered by insured depositors. Eventually, the competition erodes the discount on the risk premium that such depositors provide. In other words, due to the competition, these banks pay insured deposits about the same interest they pay uninsured players. Hence, the presence of active insured depositors does not alleviate the burden of payments on banks that face a substantial risk of default. In fact, it does not contribute at all to bank stability. At the same time, due to the ability to raise cheap funding from insured depositors at regular times, banks enter the crisis with riskier portfolios, making them more likely to fail. Hence, the presence of active insured depositors in the market leads to higher failure rates than we would have found in their absence.

The paper makes a few contributions. First, it contributes to the policy debate about deposit insurance and specifically about the potential exploration of this system by players actively seeking to earn higher returns by funding banks exposed to high risk of failure. By that, they weaken the discipline imposed on banks by their depositors and increase the expected losses of the FDIC from events of bank failure. Regulators have been studying this issue mostly regarding restrictions on brokered deposits (see Barth et al. (2020)). Such brokering made it easier to funnel insured deposits to banks at risk by diminishing search and transaction costs. To the best of my understanding, recent development in Fintech threaten to do just that, thus making the issue of mobility of insured deposits more acute.

The paper contributes to this discussion in two ways. One, it demonstrates that regulation on the mobility of insured deposits is ineffective unless it can keep the pool of mobile insured funds below a certain threshold. More specifically, the quantity of funding banks can raise through this channel should be below the demand for funds by banks that face the risk of default during good times. Failing to leave the pool below means that from the bank's perspective, absent in times of crisis, retention of the funds provided by uninsured players is not a critical factor in deciding how much risk to bear. Since few and typically not large banks generate demand for insured deposits in good times, keeping the pool of mobile insured deposits minimal is essential. That further emphasizes that minor changes in the search and transaction costs, originating from new technology, may dramatically impact the role of deposit insurance in the banking system and bank stability in general.

Second, the paper improves our understanding of bank runs by exploring the role of a feedback loop between interest rates on liabilities and the probability of bank failure. I argue that this mechanism can create a pattern that empirically resembles a coordination-driven bank run of the type discussed in the rich literature originating from Diamond and Dybvig (1983). That is, we may see banks that appear solvent and liquid are experiencing a deposit outflow that leads to their failure. The reason is that the current rates, according to which solvency is determined, do not reflect the future trajectory of rates given the bank fundamentals. Such a deposit outflow has different traits and implications than one originating from a coordination failure. Among other things, it is the unique outcome determined by the bank fundamentals, irrespective of factors such as the precision of the signal about these fundamentals or depositors' heterogeneity.

In this context, the paper critiques a third-run concept - the one suggested by Egan et al. (2017). They use a similar framework in which the interest rate interacts with the probability of failure. They argue that this framework implies multiplicity and that a bank run is a random transition from a good equilibrium, where banks pay low rates and are less likely to default, to an inferior equilibrium, with high rates and a greater likelihood of bank failure. I claim that their concept of a bank run is based on a concept of imperfect equilibrium and, as such, hinges upon assuming that banks do not have rational expectations about the behavior of depositors off the equilibrium path. The claim is inspired by the arguments that deny the existence of a similar type of multiplicity appearing in the sovereign default literature (for instance, Auclert and Rognlie (2016)).

Lastly, the paper demonstrates how the presence of a small group of risk-tolerant investors can exacerbate the severity of systemic crises. Such investors' presence lowers institutions' funding costs at the risk of default only in good times. Thus, it leads to more risk-taking but does not contribute to mitigating failure in systemic distress. This mechanism is at play when the group of risk-tolerant investors is large enough to answer the funding needs of players who face heightened risk in good times but falls short of answering the funding needs of distressed players in a crisis. Such a scenario is likely if, for instance, players need to pay a fixed cost to join the group of active risk-tolerant players. For example, we may think of depositors who need to pay a fixed price to join a mailing list that provides details of banks that offer better rates on their deposits. Depositors will be willing to pay this cost if they can often benefit from the high returns distressed banks offer. However, this is possible only if the funds held by active risk-tolerant depositors far exceed the funds distressed banks need in good times. It seems reasonable that a similar logic prevails in other financial circumstances, for instance, in asset markets with substantial information asymmetry, where a limited supply of funds is held by players who are more risk-tolerant due to being informed.

The paper is constructed as follows: In Chapter 2, I describe the model. In Chapter 3, I solve it in two stages. First, I study a simplified version of the model in which the share of banks at risk of failure is fixed. Through that, I characterize the equilibrium and explore the impact of active insured depositors on bank failures from mild aggregate shocks. Specifically, I study when the increase in failures due to the moral hazard problem they generate dominates the decline from the availability of cheap funding to banks at risk of default. Alongside, I show that the multiplicity of equilibria in the model, similar to the one that underpins the concept of bank runs advocated by Egan et al. (2017), hinges upon assuming suboptimal behavior of depositors off the equilibrium path. Then, I study the full, stochastic version of the model. I characterize the number of bank failures with active insured depositors and without them for varying levels of aggregate shock. I demonstrate my main result - such depositors raise the number of bank failures when a sizeable aggregate shock hits the banking system. In chapter 4, I conclude.

2 Model

2.1 Environment

The economy is populated by a continuum of size 1 of identical banks and a continuum of size $1 + \epsilon$ of depositors. We assume that η depositors are insured, while $\gamma + \epsilon$ are uninsured, where $\gamma = 1 - \eta$. We will assume that each bank has η insured depositors and γ uninsured depositors assigned to it as their default bank at the beginning of time.

Both banks and depositors are maximizing expected returns. All players have an outside option of investing in risk-free assets (government bond) that yields a return of $\bar{R}$. We assume that ϵ uninsured depositors are assigned to using this technology by default at the beginning of time.

2.1.1 Banks

A bank has technology which allows it to undertake a single project (limited capacity to diversify) by investing \$1. Some projects yield an expected return higher than $\bar{R}$. The bank does not have funds, but it holds a \$1 raised from its default depositors' money. The depositors have the right to liquidate their funds. The bank can prevent depositors from running or raising further funds by promising to pay back an interest of R_l on deposits, conditional on not defaulting.

The economy is subject to external shocks that hit $\delta \sim \Delta(0, 1)$ of the existing projects. We will refer to banks whose projects are hit by the shock as distressed, and we will denote the state of the bank by $s \in \{d, n\}$ (distressed or not).

The projects available to the bank differ in their risk level, which is embedded in the variable x. x is a proxy for the project's return in a crisis. Riskier projects (lower x) yield higher returns conditional on not incurring a negative shock. Accordingly, the return of a bank from investing in a project x conditional on being in a state s will be:

$$R_i(x,s) = \left\{ \begin{array}{ll} \bar{R} + \alpha + G(\bar{R} - x) & n \\ x + \alpha\xi_i & d \end{array} \right\}$$

That is to say - projects that are not hit by a shock have a deterministic return of $\bar{R}+\alpha+(\bar{R}-x)^{\theta}$. We assume that more risk yields higher profits in the good state and that the marginal return on risk is marginally decreasing, so that: $G'() > 0$, and $G''() < 0$.

In contrast, distressed projects yield a stochastic return embedded in the variable ξ. ξ is a random variable that is uniformly distributed over $[0, 1]$. It represents uncertainty about the severity of the shock for each specific project.

α is a constant reflecting the relative importance of the shock, ξ, compared to x. As we shall see later, α also determines the impact of an increase in the rate paid for deposits, R_l, on the bank's likelihood of survival. Specifically, a higher α means a weaker impact, and $\alpha = 1$, means that a one percent increase in the rate of deposits increases the likelihood of default by 1%. I assume that $\alpha > 1$, and I will show later that the assumption is essential for ensuring the model has a solution.

We assume that projects that do not get hit by a negative shock always outperform the ones that do. We embed this assumption in $G()$ by requiring that:

$$\bar{R} + G(0) > \max\{X + \alpha\xi\} = \bar{R} + \alpha \implies G(0) > \alpha$$

Also, to ensure an interior solution, we impose the Inada condition:

$$lim_{z \longrightarrow 0} G'(z) = \infty, \quad lim_{z \longrightarrow \underline{x}} G'(z) = 0$$

.

To ensure that banks attain a return greater than $\bar{R}$, we let them achieve it by picking the safest portfolio: $x = \underline{x}$. This portfolio yields α in good times and no less than 0 in bad times. Hence, a sufficient condition will be:

$$\int_0^1 (1-\delta)\alpha + \delta * 0 d\Delta = \int_0^1 (1-\delta)\alpha d\Delta > \bar{R}$$

After the returns of the project are realized, banks pay back the promised interest to depositors and earn a profit of:

$$\pi = R_i - R_l$$

If a bank cannot pay back, namely, if $R_i - R_l < 0$, it is forced to default. A bank that defaults incurs a penalty of $-\phi$, representing losing the future profits from its operations. If a bank defaults, the project is forced into liquidation and loses a share of c of its value.

2.1.2 Depositors

Each depositor will be endowed with \$1. The depositor may invest its funds with a bank in return for a promise to be paid R_l. If the bank does not default, the promise will be kept. Insured depositors get R_l if the bank defaults, while uninsured depositors get 0. In contrast, some that the government pays insured deposits back from a tax, τ, imposed on all depositors equally. Since depositors are risk neutral, the expected tax does not influence their decision-making and will not be included in the analysis).

Each depositor chooses whether to invest its funds with a distressed bank, a non-distressed bank, or by using the outside option that yields $\bar{R}$. We will assume that depositors remain with their default investment vehicle if doing so is consistent with their choices.

2.1.3 Dynamics

The economy unfolds in 3 periods:

- **Period 0**:

  [label=]Banks choose a project, x.

1• **Period 1**

[label=]The state of the economy is realized, and δ projects incur a negative shock. Banks, wholesale depositors, and sophisticated households trade in the liabilities market. Banks that fail to raise funds default.

3. **Period 2**

[label=]Project returns (ξ) are realized. Distressed banks choose whether to default. Payments are made.

I do not allow distressed banks to liquidate their project for a return of $x + \alpha\mathbb{E}[\xi]$. We may assume that this is the case, for instance, since each bank gets a signal about ξ and loses selection, prevents the marm operating (note that a signal about ξ is consistent with not choosing to default, and with raising funds from depositors at the market rate (and no higher)).

2.2 Parametric Assumptions

We will make the following parametric assumptions:

1.

$$\alpha > 1$$

As we shall see later, the assumption implies that a one-dollar increase in the interest paid to depositors, R_l, increases the probability of default by strictly less than 1.

2.

$$\underline{x} = \sqrt{4\bar{R}\alpha} - \alpha$$

The assumption assures that $R_l(\delta, x)$ is well-defined on the range $[\underline{x}, \bar{R}]$

3.

$$\alpha > \bar{R}$$

implies that the range $[\underline{x}, \bar{R}]$ is not empty, that is:

$$\underline{x} = \sqrt{4\bar{R}\alpha} - \alpha < \bar{R}$$

2.3 Bank's Problem

All banks will pay at least $\bar{R}$ for their deposits. Any lower amount will not allow them to retain depositors and prevent them. Moreover, non-distressed banks will pay exactly $\bar{R}$. If they pay more, investing with them strictly dominates the use of external storage technology. In that case, we will have an excess demand of ϵ for bank deposits. An immediate implication is that distressed banks will always pay a (weakly) higher rate for their deposits.

By construction, $\bar{R} + G(\bar{R} - X) > \alpha > 0$, so that non-distressed banks are always solvent. As for distressed banks, they will become insolvent if and only if:

$$x + \alpha\xi - R_l < 0 \implies \quad \xi \leq \frac{R_l - x}{\alpha}$$

Which will happen with probability:

$$Pr(\xi \leq \frac{R_l - x}{\alpha}) = \frac{R_l - x}{\alpha}$$

(Note that by our assumptions: $R_l \geq \bar{R} \geq x$ So that this probability is always well defined).

Thus, we can write the bank's problem as follows:

$$\max_{x \in [\underline{x}, \bar{R}]} \{ \int_0^1 [(1-\delta)(G(\bar{R} - X)) + \delta[\frac{R_l(x) - x}{\alpha} * -\phi + \int_{\frac{R_l(x)-x}{\alpha}}^1 x + \alpha\xi - R_l(x) d\xi] d\Delta]\} \quad (1)$$

Where the third expression uses the fact that $f(\xi) = 1$ on $[0, 1]$.

2.4 Equilibrium Definition

We will assume a symmetric equilibrium in which all banks choose the same risk level, x.

To simplify our work, I assume that sophisticated households choose banks over the outside storage option if both offer a return of $\bar{R}$.

An Equilibrium will consist of the following:

1. Bank's choice of risk level, $x^* \in [\underline{X}, \bar{R}]$.

2. The interest paid on distressed banks liabilities: $R_l(\delta, x) : [0, 1] \times [\underline{x}, \bar{R}] \longrightarrow \mathbb{R}$

3. For each state of the bank, $\{d, n\}$, the measure of insured and uninsured depositors funding it.

That satisfies for any state δ:

1. Given $R_l(\delta, x)$, the bank's policy solves the bank's problem (eq. 1).

2. Given $x^*, R_l(\delta, x)$, wholesale (uninsured) deposits invest in distressed banks only if they get compensated for the risk:

$$R_l(\delta, x^*) * \underbrace{(1 - \frac{R_l(\delta, x^*) - x^*}{\alpha})}_{\text{Prob. solvency}} = \bar{R} \tag{2}$$

3. Insured depositors invest in banks that pay the maximal interest. Thus, they invest in non-distressed banks only if $R_l(\delta, x^*) = \bar{R}$

4. Market clearing.

3 Solving the Model

In this section, we solve a simplified version of the model in which δ is a fixed point. The version is merely an instance of the general model in which the support of Δ is a Singleton. The results we attain will set the ground for the analysis of the stochastic case.

Since δ is fixed, we will simplify the notation and write:

$$R_l(x) := R_l(\delta, x), \quad R_l'(x) := \frac{\partial R_l(\delta, x)}{\partial x}$$

3.1 Liabilities Market - Deterministic Case

We will have δ distressed banks, each raising $(\eta + \gamma)$ dollars in the liabilities market. The banks will be able to raise those funds solely from insured depositors if and only if:

$$\eta > \delta * (\gamma + \eta) \implies \eta > \frac{\delta\gamma}{1-\delta}$$

We will divide our analysis into two scenarios depending on whether η satisfies this condition (and while ignoring the knife-edge case). The two scenarios will significantly differ, allowing us to highlight the impact of insured investors on pricing and risk.

3.1.1 Case 1: $\eta > \frac{\delta\gamma}{1-\delta}$

In this case, the supply provided by sophisticated households is large enough to fully answer the demand:

$$\eta > \frac{\delta\gamma}{1-\delta} \implies \eta > \delta * (\gamma + \eta)$$

The interest paid on liabilities of distressed banks will be given by:

$$R_l(x) = \bar{R}$$

As any higher rate leads to excess demand. Note that the interest will not depend upon the bank's choice of x. It also immediately follows that:

$$R_l'(x) = 0$$

Since the wholesale depositors' optimality condition (eq. 2) is not satisfied, distressed institutions will raise funds only from sophisticated households. In other words, we will have an inflow of insured depositors into distressed institutions and an outflow of uninsured depositors who will move to banks that did not incur the shock or invest in the storage technology. This is consistent with the empirical literature documenting an outflow of uninsured deposits and an inflow of insured depositors from distressed banks (See, for instance: Puri, 2019).

3.1.2 Case 2: $\eta < \frac{\delta\gamma}{1-\delta}$

In this case, distressed banks must raise some funds from uninsured depositors. Since some, but not all, wholesale depositors will invest with distressed banks, the wholesale depositors' optimality condition will hold with equality:

$$R_l(x^*)Pr(\text{bank is solvent}) = \bar{R}$$

Which we can write as:

$$R_l(x^*)(1 - \frac{R_l(x^*) - x^*}{\alpha}) = \bar{R} \tag{3}$$

Since in equilibrium $x^* < \bar{R}$ (interior solution) and $R_l(x^*) > \bar{R} \implies \frac{R_l(x^*)-x^*}{\alpha} > 0$, distress banks compensate depositors by the risk by paying a strictly higher rate: $R_l(x^*) > \bar{R}$. Hence, households will find it optimal to invest their money only in distressed banks, and again we will see an inflow of insured depositors replacing uninsured depositors who will leave the bank.

Multiple solutions to equation 3 might exist. The multiplicity reflects that a higher interest rate implies higher compensation to depositors as well as a higher likelihood of default. That is to say; we may have one equilibrium with low rates and another with high rates that increase the likelihood of default while compensating depositors for the added risk generated.

As we shall immediately see, we can rid ourselves of the multiplicity by assuming that the equilibrium is perfect. More specifically, we will make a single assumption:

Assumption 1. *There are no two projects $x_1, x_2 \in [\underline{x}, \bar{R}]$ such that project x_1 is more risky ($x_1 < x_2$), and yet choosing it may yield a higher expected net return when the bank is distressed:*

$$x_1 < x_2 \implies \mathbb{E}[x_1 - R_l(\delta, x_1) + \xi_i] \leq \mathbb{E}[x_2 - R_l(\delta, x_2) + \xi_i], \forall \delta \in [0, 1]$$

The state of affairs in Assumption 1 is possible only if the bank that picks a riskier project, x_1, ends up paying a lower interest rate on liabilities:

$$R_l(\delta, x_1) < R_l(\delta, x_2)$$

It is as if choosing x_2, the safer project, coordinates depositors to transition to an equilibrium with a higher interest rate and a higher likelihood of default. This does not feel intuitively right, as the movement of the bank towards a more prudent behavior is the thing that triggers depositors to make this transition. Further, the assumption violates the optimality of choosing x_2 for the bank, as it pays less than x_1 both when the bank is faring well or when the bank is under distress. Thus, in equilibrium, a bank will never choose x_2. A portfolio that leads to a high interest and risk of failure might be a part of equilibrium only if banks have some misguided belief about $R_l(\delta, x)$ in case they move to a riskier portfolio. That is, it will require a violation of equilibrium perfection.

That explains why the sovereign default literature that relies heavily on a similar framework to the one presented here, the Eaton and Gersovitz (1981) model, disregards such multiplicity. It does not arise when we solve such models using a recursive formulation since such formulation imposes optimality in each state of the model and hence enforces "perfection".

The following lemma establishes that once we implicitly impose perfect equilibrium through Assumption 1, we do not have transitioning between the low rate and low failure rate equilibrium and the high rate high failure rate equilibrium:

Lemma 3.1. *If assumption 1 holds, than:*

1. $R_l'(\delta, x) := \frac{\partial R_l(\delta,x)}{\partial x}$ *is well defined on the range* $[\sqrt{4\bar{R} - \alpha}, \bar{R}]$.

2. *If* $\eta < \frac{\delta\gamma}{1-\delta}$, *then* $R_l'(\delta, x^*) < 0$; *Else* $R_l'(\delta, x^*) = 0$.

3. $R_l'(\delta, x^*)$ *is convex in* x, *or:* $R_l''(\delta, x^*)$

4. *If* $\eta < \frac{\delta\gamma}{1-\delta}$, *then* $-R_l'(\delta, x^*) > \frac{\bar{R}}{\alpha}$

For the full proof, see the appendix.

Corollary 3.1.1. *When assumption 1 holds, the unique interest rate in equilibrium is either* $\bar{R}$ *or:*

$$R_l(\delta, x^*) =_{R_l} \{R_l * (1 - F((\frac{R_l - x - \phi}{\alpha}) = \bar{R}\}$$

The corollary implies that the solution of Equation 3 that consist of a high-interest rate and high probability of default is not a part of an equilibrium. The reason for that is hidden in the proof of lemma 3.1. As it turns out, when we are in the high rate environment, a choice of a riskier portfolio (a lower x) results in a lower interest rate on the bank liabilities and a lower probability of default. This is because when rates are high, they are more responsive to an increased probability of failure. Hence, when the rate increases, it increases the likelihood of failure, which requires a much higher rate to compensate, etc. This process never converges. Instead, when the bank chooses a riskier portfolio, rates will decline dramatically to increase its expected return in times of distress. That will imply a lower probability of failure, supporting the higher rates.

That is to say, a situation in which the bank chooses a x for which equation 3 has two solutions, and the one with the higher rate is selected cannot be a part of a perfect equilibrium. Whenever the rate is the higher solution to the equation, and assuming that depositors are expected to act rationally off-path, the bank can be strictly better off by choosing a riskier portfolio.

This implies that papers such as Egan et al. (2017) that regarded bank runs as a sunspot-driven transition from a low-rates-low-probability-of-default equilibrium to a high-rates-high-probability-of-default relies on a somewhat less compelling notion of equilibrium. To employ such a notion, they must either violate the off-path optimality of depositors or provide a compelling story about what happens when a bank that pays a higher interest rate chooses to take more risk and why this occurrence deviates from what is prescribed by the equilibrium conditions.

Thus, a sound limited commitment model of the type presented here cannot, in my view, accommodate bank runs. Doing so will require a richer framework which, to the best of my knowledge, was yet developed. However, this framework can incorporate something empirically similar to a run, although it is driven by a different mechanism. It implies that a bank may seem solvent and liquid and yet lose all of its funding. However, this will not be the result of a coordination failure. Instead, it is because the interest rates that persist before the bank enters a state of distress do not reflect the actual price of liabilities it will pay once the state is realized. The distress and the interest enter into a feedback loop, reinforce each other, and may lead a bank that seems highly likely to be able to pay its liabilities at its current prices to end up becoming insolvent.

3.2 Bank's Problem - Deterministic Case

Lemma 3.2. *Fix δ. Banks' optimality condition is given by:*

$$(1-\delta)G'(\bar{R}-x) = \delta[1 - \frac{R_l'(\delta,x)}{\alpha} f(\frac{R_l(\delta,x)-x)}{\alpha})\phi + (1-R_l'(\delta,x))(1-F(\frac{R_l(\delta,x)-x}{\alpha}))] \quad (4)$$

In the expression above, the LHS is the marginal return (in the good state) from lowering x. The RHS is the marginal cost. Beyond the direct impact of x on the banks' payoff, it also indirectly determines the rates required to compensate depositors if the bank is distressed, $R_l(\delta, x)$. Specifically, lowering x diminishes the expected return in such a state by $1 - R_l'(x)$.

The expression $\frac{R_l'(\delta,x)-1}{\alpha} f(\frac{R_l(\delta,x)-x)}{\alpha})$ embeds the cost of lower x in terms of higher likelihood of insolvency. The expression $\frac{R_l'(\delta,x)-1}{\alpha}$ captures the impact of x on the needed value of ξ for preventing insolvency, while the expression $f(\frac{R_l(\delta,x)-x)}{\alpha})$ is the marginal impact of that additional ξ on the likelihood of remaining solvent.

As mentioned, $(1 - R_l'(\delta, x))$ embeds the contribution of x to the expected payoff of the bank in times of distress. Since the gain is received by the bank only if it manages to stay solvent, we multiply it by the likelihood of remaining solvent, $(1 - F(\frac{R_l(\delta,x)-x}{\alpha}))$, to get the desired result.

Using the two expressions, we can show that as long as the penalty from default is high enough, banks choose riskier portfolios when they have access to cheap funding from insured deposits.

Lemma 3.3. *Fix the probability of a bank being distressed, δ. If the penalty that the bank incurs for default, ϕ, is greater than $\frac{\alpha^2}{\bar{R}}$, then in the presence of insured depositors bank will pick a riskier portfolio, $\tilde{x}$, compared to the portfolio that they will choose in their absence, x^*. Or:*

$$\phi > \frac{\alpha^2}{\bar{R}} \implies \tilde{x} < x^*$$

For the complete proof, see the appendix.

3.3 Default Rates

In this section, I study the impact of the level of η on default rates. The access to cheap funding from insured depositors reduces the burden imposed on distressed institutions from the liabilities market. By that, it increases their likelihood of survival. At the same time, as demonstrated in Lemma 3.3, cheap funding induces banks to hold riskier portfolios. As we shall immediately see, the direction of the impact on failure rates will be determined by the strength of the incentives to take further risks.

For this part of the analysis, I fix δ and compare two scenarios related to the share of non-sticky insured depositors: one in which the measure of insured deposits is sufficient to answer the needs of distressed banks ($\eta < \frac{\gamma\delta}{1-\delta}$), and the other where it is not. The main result is as follows:

Theorem 3.4. *Fix $\delta = \delta^*$, and let $\rho(\delta^*, \eta)$ be the measure of bank failures in state δ^* when the measure of sophisticated households is given by η. Than, $\rho(\delta^*, \eta_1) > \rho(\delta^*, \eta_2) \quad \forall \eta_1 > \frac{\delta^*\gamma}{1-\delta^*} > \eta_2$ if and only if:*

$$\frac{G'(R_l(\delta^*, x_n^*) - x_n^*)}{G'(\bar{R} - x_n^*)} > \frac{1}{1 - R_l'(\delta^*, x_n^*)} \tag{5}$$

Where x_n^* denotes the risk level at the equilibrium without sufficient insured depositors.

Corollary 3.4.1. *There exists a threshold level, $\lambda < 0$, such that if $G''() < \lambda$ on the range $[\hat{x}, x^*]$, $\rho(\delta, \eta_1) > \rho(\delta, \eta_2) \quad \forall \eta_1 > \frac{\delta\gamma}{1-\delta} > \eta_2$*

The theorem states that two forces determine the relationship between the default rates in both cases. On the one hand, the cost of picking a riskier portfolio (lowering x) declines from a $1 - R_l'(\delta, x^*) > 1$ to just 1. When insured depositors answer the demand for liabilities, picking a riskier does not indirectly diminish the bank's returns by increasing the interest it needs to pay on its liabilities. The stronger the decline in the cost of risk, the stronger the incentive to lower x. However, this incentive need not be strong enough to induce banks to increase their risk to generate a lower *net* return on the bad state. Banks will do so only if increasing their risk provides sufficient compensation. That will be the case when the return from more risk-taking is not declining too rapidly.

It is interesting to see that if the return on risk-taking (lowering the expected yield in distress by \$1) is not changing much, banks will always aim for a lower expected return in times of distress, and default rates will increase (see corollary). In this context, note that with sufficient insured deposits, banks can ensure the same expected return in the bad state while picking higher risk levels (lower x). Not doing so implies that the bank chooses to spread the premium it got from discounted liabilities for boosting its returns in both states of the world rather than only in the good state.

Note that the result might also have implications about the impact of the initial level of x_n^* on the failure rates. I need to explore it further (all I have for now is the finding that $R_l'(x_n^*)$ is strictly decreasing in x_n^*, which improves the likelihood of the condition being satisfied for risky banks; However, for these banks, $R_l(\delta, x_n^*)$ is also greater, which means there is greater room for a significant decline in the marginal contribution of additional risk.

3.4 Stochastic Case

Now, I turn to the full model in which the aggregate state of the banking system, captured by the share of banks in distress, δ, is stochastic. I fix the share of insured depositors, η, and examine how the market behaves when the banking system is hit by shocks of varying sizes.

3.5 Price of Liabilities

Since the pricing of liabilities occurs after δ is revealed, the analysis is almost identical to the one appearing in section 3.1. When the distressed banks' demand for funds can be fully answered by insured depositors ($\eta > \frac{\delta\gamma}{1-\delta}$):

$$R_l(\delta, x^*) = \bar{R}, \quad \frac{\partial R_l(\delta, x^*)}{\partial x} = 0$$

When this is not the case, using assumption 1, we find that:

$$R_l(\delta, x^*) =_{R_l} \{R_l * (1 - F((\frac{R_l - x - \phi}{\alpha}) = \bar{R}\}$$

Or, using the structure of $F()$:

$$R_l(\delta, x^*) = \frac{1 + \frac{x^*}{\alpha} - \sqrt{(1 + \frac{x^*}{\alpha})^2 - \frac{4\bar{R}}{\alpha}}}{2/\alpha}$$

With the derivative:

$$R_l'(\delta, x^*) = \frac{2}{\alpha^2}[1 - \frac{1 + \frac{x^*}{\alpha}}{\sqrt{(1 + \frac{x^*}{\alpha})^2 - \frac{4\bar{R}}{\alpha}}}]$$

Which is negative and strictly decreasing in x^*.

Thus, in our setting, δ will impact prices (only) by determining whether distressed banks will need to raise funds from wholesale depositors. If they will, the wholesale depositors' optimality condition (eq. 2 will bind. That, in turn, will increase R_l and render it responsive to x. Besides this channel, δ does not influence the prices (in this context, recall that we implicitly banned using δ as a coordination device to transfer between equilibria with different pricing levels in 1).

Also, by the same reasoning presented in the deterministic model, we will see an outflow of uninsured deposits from distressed banks and an inflow of insured deposits that supplement them.

3.6 Bank's Problem

The bank chooses x before the realization of δ. Its problem is:

$$\max_x \{ \int_0^1 \left[(1-\delta)G(\bar{R} - x) + \delta[F(\frac{R_l(\delta, x) - x}{\alpha}) * -\phi + \int_{R_l(\delta^*, x) - x}^1 (x + \xi - R_l(\delta, x)) f(\xi) d\xi)] d\Delta \right] \} \tag{6}$$

Taking FOC by x we get:

$$\int_0^1 (1-\delta)G'(\bar{R} - x^*) d\Delta = \int_0^1 \delta(1 - R_l'(\delta, x^*)[\frac{\phi}{\alpha} + (1 - \frac{R_l(\delta, x^*) - x^*}{\alpha})] d\Delta \tag{7}$$

Lemma 3.5. *Let $x^*(\eta)$ denote the equilibrium value of x given that η depositors are insured. If $\phi > \frac{\alpha^2}{\bar{R}}$, then for any $\eta_1, \eta_2 \in [0,1]$, $\eta_1 < \eta_2 \implies x^*(\eta_1) > x^*(\eta_2)$*

The theorem states that an increase in the measure of insured depositors leads banks to pick a riskier portfolio. Using it, we can now characterize default rates.

3.7 Default Rates

Let $\rho(\eta, \delta)$ denote the default rates with η insured depositors, given that the aggregate state is δ. Also, define a function, μ:

$$\mu(\eta_1, \eta_2, \delta) = \rho(\eta_2, \delta) - \rho(\eta_1, \delta)$$

μ finds the impact of transitioning from η_1 to η_2 insured depositors for each value of δ. $\mu(\eta_1, \eta_2, \delta) > 0$ implies that the transition to η_2 increased failures when the aggregate state is δ.

Theorem 3.6. *For any pair $\eta_1 < \eta_2$, we find that:*

$$\mu(\eta_1, \eta_2, \delta) = \left\{ \begin{matrix} > 0, & \textit{for } \delta \leq \eta_1 \\ \lesseqqgtr 0, & \textit{for} \delta \in (\eta_1, \eta_2] \\ > 0, & \textit{for } \delta > \eta_2 \end{matrix} \right\}$$

Let $\mu(\delta, \eta) : [0, 1] \times [0, 1] \longrightarrow [0, 1]$ be a mapping from the state of the economy and the measure of sophisticated households to the measure of bank defaults. Also, define $\mu(\tilde{\delta, \eta}) = \mu(\delta, \eta) - \mu(\delta, 0)$ - that is, for each state, δ, it provides the difference in the measure of failures between a scenario in which there are no sophisticated households and the state in which there are η sophisticated households. We plot its behavior for the case in which:

$$F(R_d(\delta, x_0^*) - x_0^* - \phi) - F(\bar{R} - x_0^* - \phi) < -R_d'(\delta, x_0^*)(1 - F(R_d(\delta, x_0^*) - x_0^* - \phi)$$

(case 1), and for the case when the condition does not hold (case 2) in Figure 1. In the graph, the x-axis symbolizes the size of the shock, while the y-axis is the difference in failure rates. In blue, we plot the difference for η_1 mobile insured depositors, and in orange for $\eta_2 > \eta_1$ mobile insured players.

We can see that the presence of the mobile insured players leads to a “jump” in failure rates

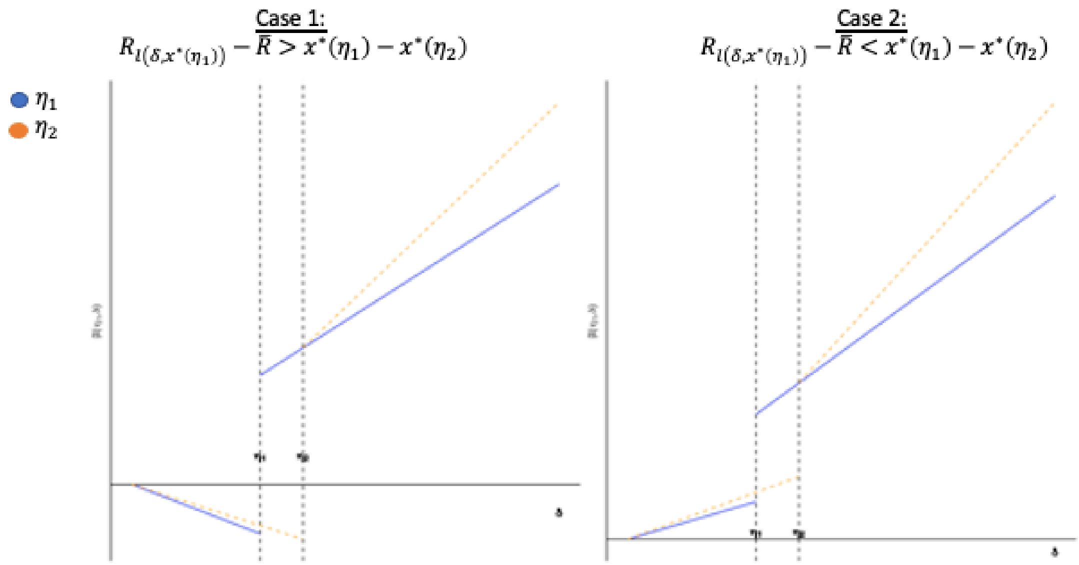

Figure 1: The difference in bank failure rates for different levels of the aggregate shock - a world with $\eta_i, i = \{1, 2\}$ mobile insured depositors versus a world with no insured depositors at all

when the aggregate shock is high enough. That is, when the demand for funding by banks in distress is higher than what these depositors can provide.

Now, imagine a world with only two states: δ_1, that happens with prob. $p >> 0.5$, and $\delta_2 > \delta_1$, that happens with probability $1 - p$. These can be thought of as a normal time, which is highly likely and sees very banks in distress, and a crisis time, which is rare and implies a greater share of banks at risk of failure. If $\eta >= \delta)1$ the funds held by mobile insured depositors are enough to answer the liquidity needs of all distressed players in the regular state. Their presence diminishes the cost of being distressed, leading to more risk-taking. However, in δ_2, this risk-taking leads to higher failure rates, as demonstrated in figure 1.

The takeaway is that very little is needed for active insured depositors to compromise the disciplinary role of bank liabilities. We require that their funds will suffice to answer the liquidity needs of distressed banks in normal times. Typically, in regular time, few banks are exposed to failure and most of them are small. Hence, A fairly small pool of funds held by such sophisticated depositors can fully meet such needs. Thus, regulation of flows of insured deposits that do not keep this pool below this low threshold is ineffective, even if it successfully blocks the majority of insured deposit flows through the banking system.

4 Conclusion

Deposit inflows significantly influence bank stability. A notable observation is that even a modest influx of funds into distressed banks can impact default rates during systemic crises. The "safety net" these depositors provide proves beneficial only when utilized by a limited number of banks. If too many banks compete for these deposits, it falls short of improving their survival prospects as expected. Yet, it leads them to enter the crisis with a riskier portfolio.

The substantial potential impact of a small group of active insured depositors sheds light on the possible effects of new regulations, technologies, and industry trends on banking stability. Specifically, it suggests that recent advancements in Fintech, which lessen the friction associated with inter-bank fund transfers, do more than increase the likelihood of deposit run-off. Instead, they redefine the dynamics of banking and the interplay between deposits and portfolio choice. Such technology, even if not widespread, can significantly affect banking behaviors by facilitating fundraising from insured depositors in the event of an idiosyncratic shock. Similarly, even the slightest loopholes in the regulatory net can undermine its effectiveness in preserving bank liabilities as a deterrent against excessive risk-taking.

A Proofs Appendix

Proof of Lemma 3.1

Proof. Let $\eta < \frac{\delta\gamma}{1-\delta}$. Then, $R_l(\delta, x)$ is defined as the value R_l solving:

$$R_l(1 - \frac{R_l - x}{\alpha}) = \bar{R}$$

Which can be written as a quadratic:

$$-\frac{1}{\alpha}R_l^2 + (1 + \frac{x}{\alpha})R_l - \bar{R}$$

The solutions will be:

$$\frac{1 + \frac{x}{\alpha} \pm \sqrt{(1 + \frac{x}{\alpha})^2 - \frac{4\bar{R}}{\alpha}}}{2/\alpha}$$

For $x \geq \sqrt{4\bar{R}\alpha} - \alpha$, the term inside of the square root is (weakly) positive. Picking the smallest solution to the equation, we get:

$$R_l = \frac{1 + \frac{x}{\alpha} - \sqrt{(1 + \frac{x}{\alpha})^2 - \frac{4\bar{R}}{\alpha}}}{2/\alpha}$$

Which is well-defined. For the case in which $\eta < \frac{\delta\gamma}{1-\delta}$, $R_l(\delta, x) = \bar{R}$ (trivially).

Taking a derivative by x of the condition defining $R_l(x^*)$ (eq. 3), we get:

$$R_l'(x^*) = \frac{-\bar{R} * -f(\frac{R_l(x^*)-x^*}{\alpha})(\frac{R_l'(x^*)-1}{\alpha})}{(1 - F(\frac{R_l(x^*)-x^*}{\alpha}))^2} \tag{8}$$

Let:

$$c := \frac{\bar{R}f(\frac{R_l(x^*)-x^*}{\alpha})}{(1 - F(\frac{R_l(x^*)-x^*}{\alpha}))^2} = \frac{\bar{R}}{(1 - (\frac{R_l(x^*)-x^*}{\alpha}))^2} \geq \bar{R} > 0$$

Where the first equality merely reflects the uniformity of F.

Rewrite eq. 8 as:

$$\frac{R_l'(x^*)}{R_l'(x^*) - 1} = c/\alpha > 0$$

If $R_l'(x^*) > 0$, then it must be the case that:

$$R_l'(x^*) - 1 > 0 \implies R_l'(x^*) > 1$$

But in that case, x^* cannot be optimal - choosing a lower value increases the expected return of the bank (and its likelihood of survival) in times of distress by $R_l'(x^) - 1 > 0$.

Thus, it must be that:

$$R_l'(x^*) < 0$$

Moreover, we can show that the derivative is negative for all $x \in (\underline{x}, \bar{R}]$. Recall, we can define $R_l(\delta, x)$ by:

$$\frac{1 + \frac{x}{\alpha} - \sqrt{(1 + \frac{x}{\alpha})^2 - \frac{4\bar{R}}{\alpha}}}{2/\alpha}$$

Taking FOC w.r.t to x, we get

$$\frac{\partial R_l(\delta, x)}{\partial x} = \frac{\frac{1}{\alpha} - \frac{1}{2}((1 + \frac{x}{\alpha})^2 - \frac{4\bar{R}}{\alpha})^{-\frac{1}{2}} * 2(1 + \frac{x}{\alpha}) * \frac{1}{\alpha}}{2/\alpha} = \frac{1}{2}(1 - \frac{1 + \frac{x}{\alpha}}{((1 + \frac{x}{\alpha})^2 - \frac{4\bar{R}}{\alpha})^{\frac{1}{2}}})$$

Which is negative iff:

$$1 + \frac{x}{\alpha} > \sqrt{(1 + \frac{x}{\alpha})^2 - \frac{4\bar{R}}{\alpha}} \implies (1 + \frac{x}{\alpha})^2 > (1 + \frac{x}{\alpha})^2 - \frac{4\bar{R}}{\alpha}$$

which always holds. Thus: $R_l'(\delta, x) < 0$.

Similarly, we can prove that $R''(\delta, x) < 0$:

$$\frac{\partial^2 R_l(\delta, x)}{\partial^2 x} = \frac{-[\frac{1}{\alpha}(1 + \frac{x}{\alpha})^2 - \frac{4\bar{R}}{\alpha})^{\frac{1}{2}} - \frac{1}{2}((1 + \frac{x}{\alpha})^2 - \frac{4\bar{R}}{\alpha})^{-\frac{1}{2}} * 2(1 + \frac{x}{\alpha}\frac{1}{\alpha}]}{(1 + \frac{x}{\alpha})^2 - \frac{4\bar{R}}{\alpha}}$$

Rearranging:

$$= \frac{\frac{1}{\alpha}((1 + \frac{x}{\alpha})^2 - \frac{4\bar{R}}{\alpha})^{\frac{1}{2}} \left(\frac{(1+\frac{x}{\alpha}}{(1+\frac{x}{\alpha})^2 - \frac{4\bar{R}}{\alpha})^{\frac{1}{2}}} - 1 \right)}{(1 + \frac{x}{\alpha})^2 - \frac{4\bar{R}}{\alpha}}$$

Since $(1+\frac{x}{\alpha})^2 - \frac{4\bar{R}}{\alpha} > 0$ on the range $[\underline{x}, \bar{R}]$, and since $1+\frac{x}{\alpha} > \sqrt{(1+\frac{x}{\alpha})^2 - \frac{4\bar{R}}{\alpha}}$, we find that:

$$R''(x) < 0$$

Finding that $R_l(\delta, x)$ is strictly decreasing in x, we can find the upper and lower bound of $-R_l(\delta, x)$. To get the lower bound of $-R_l(\delta, x)$, we plug in $x = \bar{R}$, and get:

$$R_l'(\delta, \bar{R}) = \frac{1}{2}(1 - \frac{1+\frac{\bar{R}}{\alpha}}{((1+\frac{\bar{R}}{\alpha})^2 - \frac{4\bar{R}}{\alpha})^{\frac{1}{2}}}) = \frac{1}{2}(1 - \frac{1+\frac{\bar{R}}{\alpha}}{((1-\frac{\bar{R}}{\alpha})^2)^{\frac{1}{2}}}) = \frac{\frac{\bar{R}}{\alpha}}{\frac{\bar{R}}{\alpha} - 1}$$

Which is negative (as desired). Similarly, plugging in $\underline{x}$, we find that:

$$lim_{x \rightarrow \underline{x}} R_l'(x) = -\inf$$

Thus:

$$R_l'(x) \in [-\inf, \frac{\frac{\bar{R}}{\alpha}}{\frac{\bar{R}}{\alpha} - 1}]$$

Further developing the expression we got above, we find:

$$R_l'(x^*) = \frac{c}{\alpha}(R_l'(x^*) - 1) \implies R_l'(x^*) = \frac{-\frac{c}{\alpha}}{1 - \frac{c}{\alpha}}$$

Which is negative if and only if:

$$1 - \frac{c}{\alpha} > 0 \implies c < \alpha$$

Plugging back c:

$$\frac{\bar{R}}{(1 - \frac{R_l(x^*) - x^*}{\alpha})^2} < \alpha$$

Which can be written as:

$$\frac{\bar{R}}{\alpha} < (Pr.(\text{solvent given x*}))^2$$

Since $R_l(\delta, x)$ is decreasing in x, the probability of solvency is minimized at $\underline{x} = \sqrt{4\bar{R}\alpha} - \alpha$. In this case, $R_l(\delta, \underline{x})$ willl be given by:

$$\frac{(1+\frac{\underline{x}}{\alpha})-0}{\alpha/2} = \sqrt{\alpha\bar{R}}$$

The probability of solvency will be:

$$1 - \frac{R_l(\delta, \underline{x}) - \underline{x}}{\alpha} = 1 - \frac{\sqrt{\alpha\bar{R} - \sqrt{4\alpha\bar{\bar{R}} - \alpha}}}{\alpha} = \sqrt{\frac{\bar{R}}{\alpha}}$$

Thus:

$$\frac{\bar{R}}{\alpha} \leq (Pr.(\text{solvent given}\underline{x}))^2 = \frac{\bar{R}}{\alpha} < (Pr.(\text{solvent given x}))^2, \qquad \forall x \in (\underline{x}, \bar{R}]$$

As desired.

Moreover, now we can prove that $R_l(x)$ is convex, that is that: $R_l''(x) < 0$. Recall:

$$R_l'(x) = \frac{-\frac{c}{\alpha}}{1-\frac{c}{\alpha}}$$

Taking derivative by x:

$$R_l''(x) = \frac{\frac{\partial c/\alpha}{\partial x}(1-c/\alpha) + \frac{\partial c/\alpha}{\partial x}(-c/\alpha)}{(1-\frac{c}{\alpha})^2} = -\frac{\frac{\partial c/\alpha}{\partial x}}{(1-\frac{c}{\alpha})^2}$$

Note:

$$\text{sign}(R_l''(x)) = \text{sign}(\frac{\partial c/\alpha}{\partial x}) = \text{sign}(\frac{\partial c}{\partial x})$$

And:

$$\frac{\partial c}{\partial x}) = \frac{0 - 2(1-\frac{R_l(x)-x}{\alpha})(\frac{-R_l'(x)+1}{\alpha})}{(1-\frac{R_l(x^*)-x^*}{\alpha})^4}$$

By $R_l'(x) < 0$, $(\frac{-R_l'(x)+1}{\alpha}) > 0$, and we already saw that the probability of the bank being solvent, $(1 - \frac{R_l(x)-x}{\alpha})$, is always positive. Thus:

$$\text{sign}(R_l''(x)) = \text{sign}(\frac{\partial c}{\partial x}) = -$$

Last, use:

$$R_l'(x) = \frac{-\frac{c}{\alpha}}{1-\frac{c}{\alpha}}$$

$$c \geq \bar{R}$$

to infer:

$$-R_l'(x) = \frac{\frac{c}{\alpha}}{1-\frac{c}{\alpha}} > \frac{c}{\alpha} > \frac{\bar{R}}{\alpha}$$

Q.E.D □

Proof of Lemma 3.2

Proof. Assuming the optimal default policy, the bank's problem (eq. 1) can be written as:

$$\pi = \max_x \{(1-\delta)G(\bar{R}-x) + \delta[F(\frac{R_l(\delta,x)-x}{\alpha}) * -\phi + \int_{\frac{R_l(\delta,x)-x}{\alpha}}^{1} (x+\alpha\xi - R_l(\delta,x))f(\xi)d\xi)]\} \tag{9}$$

Using integration by parts on the last part of the equation:

$$\int_{\frac{R_l(\delta,x)-x}{\alpha}}^{1} (x+\alpha\xi - R_l(\delta,x))f(\xi)d\xi =$$

$$[(x+\alpha\xi - R_l(\delta,x))F(\xi)]|^{1}_{\frac{R_l(\delta,x)-x}{\alpha}} - \int_{\frac{R_l(\delta,x)-x}{\alpha}}^{1} F(\xi)\alpha d\xi =$$

$$x+\alpha - R_l(\delta,x) - \alpha\int_{\frac{R_l(x)-x}{\alpha}}^{1} F(\xi)d\xi$$

Plugging back into the bank's problem:

$$\pi = \max_x \{(1-\delta)G(\bar{R}-x) + \delta[F(\frac{R_l(\delta,x)-x}{\alpha}) * -\phi + x + \alpha - R_l(\delta,x) - \alpha\int_{\frac{R_l(x)-x}{\alpha}}^{1} F(\xi)d\xi)]\} \tag{10}$$

Taking FOC:

$$(1-\delta)G'(\bar{R}-x) = \delta * \frac{R_l'(\delta,x)-1}{\alpha} f(\frac{R_l(\delta,x)-x}{\alpha}) - \phi + 1 - R_l'(\delta,x) - \alpha\frac{d}{dx}\int_{\frac{R_l(x)-x}{\alpha}}^{1} F(\xi)d\xi)]$$

Using Leibniz's Rule:

$$\alpha \frac{d}{dx} \int_{\frac{R_l(x)-x}{\alpha}}^{1} F(\xi) d\xi) = \alpha \frac{d \frac{R_l(\delta,x)-x}{\alpha}}{dx} F(\frac{R_l(\delta,x)-x}{\alpha}) = (R_l'(x)-1)F(\frac{R_l(\delta,x)-x}{\alpha})$$

Plugging back into the bank's problem:

$$(1-\delta)G'(\bar{R}-x) = \delta[\frac{R_l'(\delta,x)-1}{\alpha} f(\frac{R_l(\delta,x)-x)}{\alpha}) - \phi + 1 - R_l'(\delta,x) - (R_l'(x)-1)F(\frac{R_l(\delta,x)-x}{\alpha}))]$$

Or:

$$(1-\delta)G'(\bar{R}-x) = \delta[1 - \frac{R_l'(\delta,x)}{\alpha} f(\frac{R_l(\delta,x)-x)}{\alpha})\phi + (1-R_l'(\delta,x))(1-F(\frac{R_l(\delta,x)-x}{\alpha}))] \tag{11}$$

□

Proof of Lemma 3.3:

Proof. Plugging in the uniform structure of $F()$ into the bank's optimality condition appearing in equation 4, we get the following expression

$$(1-\delta)G'(\bar{R}-x^*) = \delta(1-R_l'(\delta,x^*)[\frac{\phi}{\alpha} + (1-\frac{R_l(\delta,x^*)-x^*}{\alpha})] \tag{12}$$

Using analogous steps to those taken in the proof of Lemma 3.2, we can solve for the case in which $\eta < \frac{\gamma\delta}{1-\delta}$. As explained, in this case the going price in the liabilities market will be: $R_l(\delta,x) = \bar{R}$ irrespective of the risk level (x). It immediately follows that: $R_l'(\delta,x) = 0$ Denoting the solution to the bank's problem in this case by $\tilde{x}$, the bank optimality condition reads:

$$(1-\delta)G'(\bar{R}-\tilde{x}) = \delta[\frac{\phi}{\alpha} + (1-\frac{\bar{R}-\tilde{x}}{\alpha})] \tag{13}$$

$$\phi > \frac{\alpha^2}{\bar{R}} \implies \frac{\bar{R}}{\alpha} > \frac{\alpha}{\phi}$$

Focusing on the LHS:

$$\frac{\alpha}{\phi} = 1 + \frac{1}{\frac{\phi}{\alpha}} - 1 = \frac{\frac{\phi}{\alpha} + 1}{\frac{\phi}{\alpha}} - 1 > \frac{\frac{\phi}{\alpha} + (1 - \frac{\bar{R} - \tilde{x}}{\alpha})}{\frac{\phi}{\alpha} + (1 - \frac{R_l(\delta, x^*) - x^*}{\alpha})} - 1$$

Where the last inequality stems from the fact that $(1 - \frac{R_l(\delta, x^*) - x^*}{\alpha})$, $(1 - \frac{\bar{R} - \tilde{x}}{\alpha})$ are the probabilities of bank survival with and without access to sufficient funds from insured depositors. Hence, they are both in the range $[0, 1]$.

Above, we saw that: $\frac{\bar{R}}{\alpha}$ is the lower bound of $-R_l'(\delta, x^*)$. Thus, we can write:

$$-R_l'(\delta, x^*) > \frac{\bar{R}}{\alpha} > \frac{\alpha}{\phi} > \frac{\frac{\phi}{\alpha} + (1 - \frac{\bar{R} - \tilde{x}}{\alpha})}{\frac{\phi}{\alpha} + (1 - \frac{R_l(\delta, x^*) - x^*}{\alpha})} - 1$$

Rearranging:

$$(1 - R_l'(\delta, x^*))[\frac{\phi}{\alpha} + (1 - \frac{R_l(\delta, x^*) - x^*}{\alpha})] > \frac{\phi}{\alpha} + (1 - \frac{\bar{R} - \tilde{x}}{\alpha})$$

This allows us to derive an equation comparing the marginal value of x at the equilibrium point for each scenario:

$$(1-\delta)G'(\bar{R}-x^*) = \delta(1-R_l'(\delta, x^*))[\frac{\phi}{\alpha}+(1-\frac{R_l(\delta, x^*) - x^*}{\alpha})] > \delta[\frac{\phi}{\alpha}+(1-\frac{\bar{R} - \tilde{x}}{\alpha})] = (1-\delta)G'(\bar{R}-\tilde{x}) \tag{14}$$

By the concavity of G, we can see that equation 14 implies:

$$\bar{R} - x^* < \bar{R} - \tilde{x} \implies \tilde{x} < x^*$$

Q.E.D

$\square$

Proof of Theorem 3.4

Proof. Let ρ^* denote the measure of failed banks in equilibrium. Let the subscript j of equilibrium object z^* denote whether it refers to the state in which there are enough insured depositors ($j = i$) or not ($j = n$). Recalling that banks fail only if $\xi \leq \frac{R_l(x^*)-x^*}{\alpha}$, we can write:

$$\rho_i^* := \delta F(\frac{\bar{R} - x_I^*}{\alpha})$$

$$\rho_n^* := \delta F(\frac{R_l(x_n^*) - x_n^*}{\alpha})$$

We will have more failures with a sufficient measure of insured depositors if and only if:

$$\rho_i^* - \rho_n^* = \delta F(\frac{\bar{R} - x_I^*}{\alpha}) - \delta F(\frac{R_l(x_n^*) - x_n^*}{\alpha}) > 0$$

$$F(\frac{\bar{R} - x_I^*}{\alpha}) > F(\frac{R_l(x_n^*) - x_n^*}{\alpha}) \implies \frac{\bar{R} - x_I^*}{\alpha} > \frac{R_l(x_n^*) - x_n^*}{\alpha} \implies x_n^* - x_i^* > R_d(\delta, x_n^*) - \bar{R} \tag{15}$$

The result reflects our assumption that the likelihood of survival of a bank can be derived from its expected revenue in times of distress. As such, it is independent of the distribution of ξ (of the structure of $F()$). It states that insurance will increase failure rates only if it leads banks to choose a portfolio that implies a lower expected revenue in distress in spite of the decrease in their interest payments. Put differently, failures will increase if and only f banks will choose to transfer to a portfolio that diminishes the return of the project in times of distress by more than $R_d(\delta, x_n^*) - \bar{R}$.

We can rewrite condition 15 as:

$$x_i^* < x_n^* + (\bar{R} - R_l(\delta, x_n^*)$$

Denote: $\hat{x}$, the expression.

$$\hat{x} = x_n^* + (\bar{R} - R_l(\delta, x_n^*))$$

This is the threshold of x such that picking a riskier portfolio (lower x) implies more defaults with sufficient insured deposits. If $x_i^* < x_n^*$, it should be the case that it will be optimal to increase the risk level at $\hat{x}$ it is optimal to increase the risk level, or that:

$$(1-\delta)G'(\bar{R}-\hat{x}) > \delta[\frac{\phi}{\alpha} + (1 - \frac{\bar{R}-\hat{x}}{\alpha})]$$

Plugging in our expression for $\hat{x}$, we get:

$$(1-\delta)G'(R_l(\delta, x_n^*) - x_n^*) > \delta[\frac{\phi}{\alpha} + (1 - \frac{R_l(\delta, x_n^*) - x_n^*}{\alpha})]$$

Substituting the bank optimality condition without access to sufficient insured deposits (equation 12) into the RHS, we get:

$$(1-\delta)G'(R_l(\delta, x_n^*) - x_n^*) > \delta[\frac{\phi}{\alpha} + (1 - \frac{R_l(\delta, x_n^*) - x_n^*}{\alpha})] = \frac{(1-\delta)G'(\bar{R}-x^*)}{(1-R_l'(\delta, x_n^*))}$$

Which can be rearranged to:

$$\frac{G'(R_l(\delta, x_n^*) - x^*)}{G'(\bar{R}-x_n^*)} > \frac{1}{1-R_l'(\delta, x_n^*)}$$

$\square$

Proof of Lemma 3.5

Proof. Denote: $x_1^* := x^*(\eta_1), x_2 := x^*(\eta_2)$. Using our characterization of $R_l(\delta, x)$, we can rewrite the bank's optimality condition in the stochastic case (7) where $\eta = \eta_1$ as:

$$\int_0^1 (1-\delta)G'(\bar{R}-x_1^*)d\Delta = \int_0^{\eta_1} \delta[\frac{\phi}{\alpha}+(1-\frac{\bar{R}-x_1^*}{\alpha})]d\Delta + \int_{\eta_1}^1 \delta(1-R_l'(\delta, x_1^*))[\frac{\phi}{\alpha}+(1-\frac{R_l(\delta, x_1^*) - x_1^*}{\alpha})]d\Delta$$

Which we can also represent by:

$$\int_0^1 (1-\delta)G'(\bar{R}-x_1^*)d\Delta =$$
$$\int_0^{\eta_1} \delta[\frac{\phi}{\alpha}+(1-\frac{\bar{R}-x_1^*}{\alpha})]d\Delta + \int_{\eta_1}^{\eta_2} \delta(1-R_l'(\delta,x_1^*))[\frac{\phi}{\alpha}+(1-\frac{R_l(\delta,x_1^*)-x_1^*}{\alpha})]d\Delta +$$
$$\int_{\eta_2}^1 \delta(1-R_l'(\delta,x_1^*))[\frac{\phi}{\alpha}+(1-\frac{R_l(\delta,x_1^*)-x_1^*}{\alpha})]d\Delta >$$
$$\int_0^{\eta_1} \delta[\frac{\phi}{\alpha}+(1-\frac{\bar{R}-x_1^*}{\alpha})]d\Delta + \int_{\eta_1}^{\eta_2} \delta[\frac{\phi}{\alpha}+(1-\frac{\bar{R}-x_1^*}{\alpha})]d\Delta \int_{\eta_2}^1 \delta(1-R_l'(\delta,x_1^*))[\frac{\phi}{\alpha}+(1-\frac{R_l(\delta,x_1^*)-x_1^*}{\alpha})]d\Delta =$$
$$\int_0^{\eta_2} \delta[\frac{\phi}{\alpha}+(1-\frac{\bar{R}-x_1^*}{\alpha})]d\Delta + \int_{\eta_2}^1 \delta(1-R_l'(\delta,x_1^*))[\frac{\phi}{\alpha}+(1-\frac{R_l(\delta,x_1^*)-x_1^*}{\alpha})]d\Delta$$

Where we inferred the inequality from our analysis in the deterministic case where we saw that:

$$\phi > \frac{\alpha^2}{\bar{R}} \implies \qquad (1-R_l'(\delta,x^*))[\frac{\phi}{\alpha}+(1-\frac{R_l(\delta,x^*)-x^*}{\alpha})] > \frac{\phi}{\alpha}+(1-\frac{\bar{R}-\tilde{x}}{\alpha}), \qquad \forall(\delta,x)$$

Thus, we find that:

$$\int_0^1 (1-\delta)G'(\bar{R}-x_1^*)d\Delta > \int_0^{\eta_2} \delta[\frac{\phi}{\alpha}+(1-\frac{\bar{R}-x_1^*}{\alpha})]d\Delta + \int_{\eta_2}^1 \delta(1-R_l'(\delta,x_1^*))[\frac{\phi}{\alpha}+(1-\frac{R_l(\delta,x_1^*)-x_1^*}{\alpha})]d\Delta$$

To satisfy the equality, the value of the LHS should decrease, or the value of the RHS should increase (or both). That will happen iff $x_2^* < x_1^*$. Q.E.D $\square$

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