# Hard Times Call for Fundamental Questions:

# On the Counter-Cyclical Behavior of Basic Research and The Impact of Transient Shocks on Long-Term Growth. \*

Assa Cohen

Sy Syms School of Business, Yeshiva University<sup>†</sup>

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#### Abstract

This paper investigates the impact of transient economic shocks on long-term growth, focusing on their role in shaping the nature of R&D investment. It provides novel evidence indicating that private sector investment in basic research is countercyclical. Specifically, it documents a substantial rise in private basic research expenditure in 2008 (20%), 2009 (35%), during the 2001 dot.com recession (20%), and the 1991 recession (55%). In line with Schumpeter's theory, the paper argues that weak demand induces firms to transition towards long-term R&D, prompting them to pursue more fundamental questions, and ultimately fostering technological leaps and long-term growth. Based on these findings, the paper proposes a novel semi-endogenous growth model with two types of R&D activities: creating new knowledge (basic research) and applying existing knowledge (applied research and development). After calibrating the model, it shows that the adverse impact of a recession on the level of R&D investment is countered by reallocating this investment to activities that have higher social value. When the latter effect dominates, downturns can accelerate long-term growth.

Keywords: basic research, R&D composition, endogenous growth, business cycles, innovation. JEL classification:: E32, O31, O32, O41, O47,

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<sup>&</sup>lt;sup>†</sup>Email: assa.cohen@yu.edu

# 1 Introduction

What is the impact of transient economic shocks on long-term growth? While this is a key question in macroeconomics, it is characterized by a contrast between theory and practice. The endogenous growth literature that became widely accepted in macroeconomics holds that growth is, to a great extent, a result of economic activity. More specifically, it originates from the creation of new technologies through R&D for the sake of generating profits. Since shocks shape activity, they should determine the future trajectory of growth. However, in practice, macroeconomic research separates the analysis of short-term economic movements (cycle) from the long-term behavior of growth (trend). By that, it implicitly assumes that in the long term, the growth trajectory is independent of the shocks the economy might face.

Perhaps the reason why the profession remains content with such tension is the lack of promising candidates that may resolve it. A straightforward interpretation of endogenous growth theory would be to argue that since downturns (booms) oppress (boost) economic activity, including R&D, they should hinder (expedite) technological development and, through it, long-term growth. However, the claim that downturns have a long lasting impact on technology, gauged by total factor productivity (TFP), has but scant evidence to support it (see Fernald (2015)). Further, there is at least one salient example of a downturn followed by highly rapid technological innovation - the Great Depression of 1929-1932. The depression was followed by very rapid technological growth that led many to designate the 1930s as one of the most fertile decades of the 20th century in terms of technological innovation (Field (2003), Alexopoulos and Cohen (2009)). An alternative theory is inspired by Joseph Schumpeter's claim that recessions are periods of substantial innovation that saw the seeds of growth in their aftermath. Supposedly, weak demand during recessions implies a lower opportunity cost for exploring new and novel technologies. The view has a strong intuitive appeal and has fascinated economists. However, a large body of literature documented that innovation activity, as measured by R&D expenditure and innovation output, gauged through patents, are cyclical and decline during downturns.

In this paper, I suggest a third route that consists of two components. *First*, I claim that short-term fluctuations not only determine the *level* of R&D investment but also shape it *nature*. I show that when this is considered, we find clear evidence in favor of a Schumpeterian

mechanism. Low returns from introducing new products lead companies to divert a larger share of their R&D to pursuing *fundamental* questions that increase their capacity to innovate in the future. I use aggregates generated from high-quality survey data collected by the US Census on behalf of the National Science Fund in the years 1986-2015 and demonstrate that the private sector investment in basic research, defined as "the systemic pursuit of knowledge without a well-defined commercial end in mind," is countercyclical. Furthermore, I document a surge in basic research activity during the 1991 recession (45% increase), the 2001 dot.com crisis (20& increase), and the 2007-2009 financial crisis (60% increase).

I argue that a transition towards addressing more fundamental questions is beneficial from a societal perspective. Firms engaging in open-ended inquiry, characteristic of basic research, are more likely to generate knowledge that benefits other market players. The resulting knowledge spillover can be substantial. For instance, Arora et al. (2017) found that approximately 79% of patent citations from a firm's scientific work are made in patents issued to other entities. Since firms do not internalize these positive externalities, the economy exhibits under-investment in such R&D endeavors. By redirecting R&D efforts towards broader questions, economic downturns alleviate the inefficiency arising from firms' focus on narrower, short-term work that is more easily appropriate. This mitigation can counteract the negative impact on technological development resulting from decreased overall investment. It may even surpass it. In such instances, a technological leap could follow an economic downturn, similar to what was observed after the Great Depression.

Second, my analysis accounts for what the ongoing trend of declining R&D productivity can teach us about the impact of transitory shocks on growth. The decline is manifested in the fact that R&D inputs are exponentially increasing while technological progress, gauged by TFP, is more or less constant (see Kortum (1997)). It implies that as technology evolves, it becomes harder to improve it <sup>1</sup>. Therefore, the return on investment in innovation, similar to the return on investment in other capital, is decreasing in its *stock*. As a result, innovation, like capital, will revert to its original trajectory after transient shocks that will pull it away from it. Intuitively, a decline in innovation during a recession means that firms do not deplete the pool of ideas as

<sup>&</sup>lt;sup>1</sup>There is a literature providing compelling evidence of such a mechanism, including Kortum et al. (1994), Jones (2009), and more recently Bloom et al. (2020))

fast as they usually do. As a result, new inventions will be easier to come by in the following periods, resulting in more R&D and more rapid innovation.

I incorporate both components into a novel semi-endogenous growth model with two types of R&D activities: creating new knowledge (basic research) and applying existing knowledge to launch other goods (applied research and development. I assume that introducing new goods depletes the pool of potential inventions and makes it harder to innovate. In contrast, creating new knowledge can potentially spill over to benefit other incumbents. The model has a unique, balanced growth path that fits the pattern we find in the data of growing investment in R&D despite an ongoing decline in its productivity. Firms in the model are facing a problem that is somewhat similar to the consumption-saving problem of a household in the neo-classical growth model: they can either conduct applies research, which results in launching blueprints in the current period, or use basic research, which will make them more productive in the future. I calibrate the model's BGP to moments characterizing the US economy, R&D, and patenting in 1986-2015. The calibration indicates that even without knowledge spillover from basic research, changes in R&D composition in response to a shock attenuate the impact of changes in the R&D level.

One key question that naturally arises is the importance of basic research conducted in the private sector when accounting for the fact that most basic research is conducted by academia with public funds. As I document below, the private sector accounts for about 25% of basic research expenditure - which is not negligible. However, beyond that, I would like to address two more reasons why we should care about the private sector's basic research.

The first reason is that even if private investment in basic research does not significantly impact innovation, it can signal changes in the nature of all privately funded R&D that do have such an effect. In this sense, investment in basic research can be viewed as an indicator of a broader trend toward long-term and open-ended innovation. This trend influences the allocation of resources among different activities within the same category of R&D composition. For instance, firms may increasingly invest in radical innovation, which involves creating novel products and production processes, rather than focusing on incremental innovation, primarily improving existing goods. Identifying such a change in data about R&D composition might be difficult, as both activities are defined as "development." It seems reasonable to assume that more significant knowledge spillovers characterize radical innovation development projects. R&D is inherently uncertain, and the more steps between the initial motivation and the final result, the more likely the outcome will diverge from the original intention, preventing firms from appropriating the returns from their efforts. In other words, we can generalize and say there is under-investment in long-term R&D and that a transient negative (positive) shock can mitigate (exacerbate) this issue.

The second reason is that basic research in the private sector is based on different knowledge, resources, and motivations compared to academic research. As a result, it addresses problems that publicly funded research in academia may be less likely to study or even be aware of. A prime example of the private sector's potential contribution to innovation can be found in the story of Rhodia Food's (Danisco) role in developing CRISPR, a gene editing technology expected to revolutionize biotechnology. Philippe Horvath, a senior scientist at Rhodia Food, investigated CRISPR in his study of the defense mechanisms of bacteria used for fermenting milk products such as yogurt, cheese, and ice cream. It interested the food industry because of its potential use in protecting bacteria used in fermentation from viruses known as bacteriophages. Hovarth research was focused on repeating DNA sequences in the bacteria genome, known as Clustered Regularly Interspaced Short Palindromic Repeat (CRISPR). It was discovered that these repeating sequences are often separated by DNA strings identical to the DNA of viruses that threaten the bacteria. Horvath's research was guided by the idea that these sequences were a part of the bacteria's immunity system. It demonstrated that if the DNA of a virus appears between two CRISPR sequences in a bacteria's DNA, then bacteria are immune to the virus (Ridley (2020)). In later work, Hovarth inserted the DNA of a Streptococcus bacteriophage between the CRISPR sequences of a Streptococcus bacterium and successfully created a resistant strain (Hamashige (????)). The work paved the way for the groundbreaking work of Doudna and Charpentier, who showed that CRISPR sequences cut genome sequences identical to those that appear between them. The finding, which earned Doudna and Charpentier the 2020 Nobel Prize, allows science to edit any gene and is likely to have numerous applications in biomedicine, agriculture, industrial biotechnology, and more.

The critical point of this example is not that privately funded research can be groundbreaking. Indeed, it often is. The point is that the interests of the private sector led it to accumulate knowledge and pursue research about questions that academia found to be less compelling. The reason that a researcher from Rhodia Food was the one to make the discovery is not a coincidence. It required having a solid motive for studying bacteria immunity systems, a motive that Rhodia Food had because of their extensive use of bacteria in producing dairy products.

### 1.1 Literature Review

The paper is a part of the literature that utilizes endogenous growth models to assess the impact of transient economic shocks on growth through technological innovation. Much of this literature adopted Romer's growth model (Romer (1990)) to explain the alleged persistence of transient shocks. Through this framework, persistence emerges, as a decline in R&D not only diminishes concurrent innovation but also reduces the knowledge base that will be used for innovation by future entrepreneurs. For example, Comin and Gertler (2006) utilized such a mechanism to account for what seemed like medium-length cycles in the data. More recently, papers such as Anzoategui et al. (2019) or Queralto (2020) suggested models that incorporate such a mechanism to explain the slow recovery from the 2008 financial crisis.

The paper contributes to this literature in two ways. First, it sets forth an endogenous growth model with different types of R&D activities to study how cycles impact growth by determining R&D composition. Second, it analyzes the impact of transient shocks using a semi-endogenous growth model that accommodates the on-lasting decline in R&D productivity. Such a model was first suggested by Jones (1995) and developed further by Kortum (1997). Both were motivated by the contrast between the implications of Romer (1990) and the fact that TFP growth rates in the US remained fairly constant despite an ever-growing increase in the inputs allocated to R&D. These papers aimed, among other things, to explain why R&D investment keeps growing despite its declining productivity. They used their models to show that this pattern emerges when the economy's expansion implies higher returns from launching new products and induces firms to continue their R&D investment even when the cost of creating new blueprints keeps rising. I embed a similar mechanism to generate constant growth in the model presented below.

Another work in the endogenous growth literature that has a strong relation to the paper is Akcigit et al. (2021), which uses an endogenous growth model of basic and applied research to study the sources of cross-variance in R&D composition in French firm-level data. They show that firms operating in the broader spectrum of industries are more likely to engage in basic research. In the authors' view, such firms can better appropriate the returns from the myriad applications of such research.

The paper contributes to a long tradition dating back to Schumpeter (2014) that emphasizes the potential role of recessions in accelerating future growth. The standard features of such theories, referred to by Saint-Paul (1993) as "opportunity cost" theories of growth, is the claim that recessions cause the economy to engage in more activities that increase productivity at the expense of current production. The most salient finding supporting such theories is that human capital accumulation is countercyclical. The discussion of human capital accumulation throughout the cycle began with Bean (1990). He claimed that the procyclical behavior of productivity could be explained by the diversion of labor from human capital accumulation and other investment-like activities to production.

This paper contributes to a long tradition dating back to Schumpeter (2014), emphasizing the potential role of recessions in accelerating future growth. Common features of such theories, referred to by Saint-Paul (1993) as "opportunity cost" theories of growth, claim that during recessions, the economy will engage in more activities that increase productivity at the expense of current production. The most salient finding supporting these theories is the countercyclical nature of human capital accumulation. The discussion of human capital accumulation throughout the cycle began with Bean (1990), who claimed that the procyclical behavior of productivity could be explained by the diversion of labor from human capital accumulation and other investment-like activities to production during booms. Since then, multiple papers have documented that schooling and enrollment in higher education, in particular, are countercyclical (for instance, Heylen and Pozzi (2007), Johnson (2013)). Other sporadic evidence indicates that weak demand induces different types of productivity-enhancing activities. For example, Cooper and Haltiwanger (1990) presents evidence from automobile plants showing that they are more likely to engage in machine replacement, which can hinder production during low sales periods.

This paper is also part of the literature on how fluctuations impact R&D investment. In contrast to the prediction of Schumpeterian growth theories, this literature provides unequivocal empirical evidence suggesting that both R&D investment and R&D output (patents) are cyclical (for an overview of the evidence, see Manso et al. (2021). Barleyy (2007) tried to reconcile the cyclical investment in R&D with its cyclical opportunity cost in a model that considers allocating inputs between R&D and production. He claimed that firms invest more during booms (less during busts) because vigorous (weak) demand implies higher (lower) profits from new inventions. By calibrating a macro model, he demonstrated that if R&D has a significant cost component that does not depend on the cycle, we get the same cyclical behavior of R&D observed in the data. Later work by Fabrizio and Tsolmon (2014) provided further evidence to support this theory, showing that R&D is more cyclical in industries with faster patent obsolescence, that is, in sectors in which market demand at the time the invention is launched has a greater impact on expected profit. Again et al. (2012) took a different route to reconcile Schumpeterian logic with the cyclical behavior of R&D. Rather than focusing on the aggregate, they sought to show that the Schumpeterian mechanism, in which weak demand boosts R&D by lowering its opportunity cost, is in fact at play. They used firm-level administrative data from France and showed that among firms not financially constrained, those that experience a greater decline in sales tend to invest more in R&D. Therefore, the tightening of financial restrictions may explain the decrease in R&D during a downturn, despite an opportunity cost pushing R&D investment upwards.

This paper belongs to a sub-strand of this literature. Instead of interpreting Schumpeter to imply that downturns should lead firms to divert inputs from production to R&D (which they do not), this strand of literature argues that downturns lead firms to reallocate resources from short-term R&D, which yields lower returns when demand is weak, to long-term R&D, which increases the productivity of short-term R&D in the future. The first paper to suggest focusing on R&D composition rather than investment level is Rafferty (2003). He used BRDIS/SIRD aggregated data and demonstrated that the ratio of research to development expenditure of firms is countercyclical. Similarly, a more recent paper by Manso et al. (2021) focuses on R&D output, that is, patents. They studied how an unexpected change in the production of a specific sector alters the patenting of the firms that belong to it. They showed that in response to an adverse shock, the share of patents of the firm that belongs to technology classes that it seldom patented in before increases. They interpreted this to imply that firms divert their R&D from exploitation, defined as the refinement of existing technology known to the firm, to exploration, defined as the pursuit of novel approaches. This paper contributes to this literature by documenting an even stronger pattern. It shows that at least one type of R&D activity that addresses fundamental questions, i.e., *basic research*, is countercyclical in *total size*. To my knowledge, this is the first paper to document such a fact. It has dramatic implications. The fact that firms allocate a larger *share* of their R&D to fundamental questions in downturns, documented in Rafferty (2003) and Manso et al. (2021), may merely reflect a state in which all R&D is cyclical with some activities responding less strongly to the cycle. If that is the case, a downturn diminishes all types of R&D efforts and ought to hinder innovation. On the contrary, this paper provides direct evidence that some kinds of R&D increase in *absolute* size during downturns. Furthermore, the types of R&D that attain more investment are precisely those with a more significant contribution to aggregate productivity. The result allows for a plausible story through which the impact of a downturn on R&D results in more substantial growth in its aftermath.

Manso et al. (2021) deserves additional discussion, as it resembles this paper more than any other in its logic. Manso et al. (2021) presents an elegant model that exemplifies how weak demand induces firms to divert their R&D from incremental improvement in existing technologies (exploitation) to the creation of new technologies (exploration), which requires more time, is characterized by higher risk, but may generate higher returns if successful. Manso et al. (2021) explores this question through a micro setting and demonstrates that, keeping the average growth rate constant, firms might be better off in a world with much volatility compared to a world in which the economy is not subject to any shocks. In this paper, I address the implications of such a mechanism through a macro-framework. Using a macro model, in which growth is endogenous, allows us to study different normative questions. Specifically, rather than asking whether firms would prefer to be in a volatile vs. stable world ex-ante like Manso et al. (2021), I ask whether the economy as a whole is better off if it has a bad *realization* of the stochastic process it is subject to (a recession) ex-post. Also, the macro model allows one to calibrate the model and test its ability to account for growth dynamics in the data.

Lastly, the paper belongs to the literature on private investment in basic research. Interest in the role of privately funded basic research in growth gained much traction following Mansfield (1984) article, which documented that an increase in private expenditure on basic research in a specific industry presages a significant increase in productivity growth in the coming years. A large body of literature provided evidence for substantial knowledge spillover from basic research, which implies underinvestment. For instance, Arora et al. (2017) shows that only 21% of patent citations of scientific papers published by a firm are citations appearing in its patents.

### 1.2 Defining Basic Research, Applied Research, and Development

Before proceeding, I would like to briefly discuss the definitions used to categorize R&D into three distinct subtypes: basic research, applied research, and Development. This categorization is widely recognized in the scientific community, and each activity is believed to have different objectives, methods, and outcomes.

Basic research involves studying the fundamental principles that govern natural phenomena. It is typically motivated by the need to attain a deeper understanding of phenomena rather than by a potential application of its findings. Its output consists of discoveries, concepts, or theories that lay the groundwork for further scientific investigation and potential new applications and uses. Examples of basic research include Shannon's Information Theory and the study of the thermoelectric properties of sodium cobalt oxide.

In contrast, applied research aims to produce new knowledge with a clear and direct application in mind. It often builds on basic research findings to improve existing systems or create new ones. Examples of applied research include the study of the properties of sodium cobalt oxide for potential use in sodium-ion batteries or applying Shannon's information theory to map the limits of error-correcting codes.

Development is systematically using knowledge generated through basic and applied research to create new products, technologies, and production processes. It bridges the gap between having knowledge and putting it to practical use. This involves designing, building, and refining prototypes, as well as scaling up production. For example, developing an error-correcting code algorithm for wifi communication.

To better understand these activities and their role in the private sector, let us revisit Horvath's work on CRISPR, as discussed in the Introduction. Horvath's study of the hypothesis that bacteria are immune to viruses if their genome sequences appear between CRISPR sequences in their DNA is a quintessential example of basic research. Its objective was to generate knowledge likely to contribute to the firm's activities. In this case, a food company was interested in findings that could help protect bacteria used for fermentation from viruses. However, at the time of the study, no one had a clear idea of using the knowledge it would gain.

Horvath's implementation of bacteriophage DNA into a Streptococcus bacteria, often used in yogurt production, is an excellent example of applied research. The purpose was to generate new knowledge by determining if inserting bacteriophage DNA between CRISPR sequences in the bacteria's DNA renders it immune. The knowledge was produced with a well-defined commercial goal: creating new and more resilient bacteria for fermentation. A development effort to follow would include, for instance, creating a prototype of such bacteria and scaling its production. This may consist of deciding which bacteriophage the bacteria should be protected from, handling unexpected consequences of manipulating the bacteria's DNA on its performance in fermentation, ensuring that the bacteria can reproduce, and streamlining and reducing the cost of further genetic manipulations.

The paper is arranged as follows: Section 2 presents the empirical analysis. It begins with an overview of the data, focusing on the BRDIS/SIRD survey data, aimed at assessing its internal and external validity. It continues to provide a general characterization of R&D composition in the private sector in the years 1986-2015 through descriptive statistics. Lastly, it demonstrates that basic research investment is countercyclical. It shows that a negative (positive) shock to GDP is correlated with an increase (decrease) in private investment in basic research. I demonstrate that the pattern persists when controlling for other macroeconomic variables or using an SVAR specification. Section 3 presents a semi-endogenous growth model with two types of research activities: basic versus applied research. I solve the model and show that it has a unique BGP in which the productivity of R&D gradually declines over time. Section 4 calibrates the model to moments that characterize macroeconomic variables, patenting, and R&D in 1986-2015. Section 5 concludes.

# 2 Empirical Findings

### 2.1 Data

#### 2.1.1 Business Research, Development, and Innovation Survey Data

The data about private sector R&D activity was obtained from aggregate statistics generated from the *Business Research, Development and Innovation Survey (BRDIS)* and its predecessor, *Survey of Innovation, Research, and Development (SIRD)*. These surveys have been collected annually by the US Census on behalf of the National Science Foundation since 1953. Their primary purpose was to construct aggregate statistics to characterize R&D and innovation activity in US corporations. They include questions about R&D expenditure, employment, management, and strategy. The surveys have changed significantly over the years. I will begin by providing a general description of BRDIS that is accurate as of 2016. Then, I will discuss the changes that occurred over the years and the ways in which they may affect the results presented here.

One of the unique characteristics of BRDIS, in comparison to other survey data, is its extensive sampling framework. This framework is based on the US Census Business Register, a comprehensive database of domestic businesses in the US, which is compiled by integrating administrative data from the Internal Revenue Service (IRS), the Economic Census, and supplementary information gathered through Census surveys. BRDIS's sample is drawn from all for-profit companies listed in the Business Register. BRDIS sampling frame consists of all for-profit firms in the Business Register.

The sampling of firms utilizes data from the Business Register on companies' reports of domestic employment and R&D expenditures in their tax filings. Consequently, companies with significant R&D activity or those with exceptionally high employment costs are always included in the sample. Other companies known to have some R&D activity are selected at random. For this purpose, companies are divided into strata according to their industry classification. The probability of a company being selected from its stratum is proportional to its size, as indicated by the number of employees. Similarly, companies with unknown R&D activity are randomly selected through the same process. Once a company is chosen, it is assigned a weight equal to the reciprocal of its selection probability for the sample. These weights are used in aggregate calculations, reflecting the belief that smaller companies included in the sample represent other similar-sized companies operating in the same industry. To prevent bias due to outliers, weights are capped at 250. To maintain consistency over time, companies are typically surveyed for at least five consecutive years.

As of 2016, the survey was administered to 45,000 firms. Among these, 5,000 firms received a full survey, while an additional 40,000 firms were given an abbreviated survey. If firms report significant R&D in the abbreviated survey, they are later sent the full survey. Although firms are not legally required to respond to the survey, the response rate is relatively high at 80.3% for all companies and 92.5% for those that reported R&D expenditure in their tax filings. The Census compensates for non-response bias by assigning companies weights that are reciprocal to the probability of non-response for companies of similar size within the same industry. Company responses are scrutinized and edited by the Census to ensure their quality in a process that often involves direct communication between Census staff and the firm. Further, for large firms with substantial R&D activity, it is common for the Census to offer guidance and assistance in completing the survey to ensure the quality of the submission. This guidance and the editing process improve the internal validity of the results.

The Census assesses that BRDIS is subject to a small sampling error. Large companies are always included, while the sample of small companies is quite large. Accordingly, they find that the standard relative error of the aggregate measures of the survey, such as total R&D expenditure or employment, is quite low at under 1%. I tend to accept this claim, and I believe that, as of 2016, BRDIS has good external validity.

The survey requires firms to provide elaborate details about their R&D expenditure. As a part of that, firms are required to break down their R&D activity along two dimensions. First, they are required to state who paid for the activity - the firm itself, the federal government, or a different entity. This clear distinction, alongside the exclusion of any government entities from the sample frame, greatly improves the likelihood that the patterns we witness are not driven by the actions of government agents (more on this below). Alongside, firms are required to report their research composition by breaking down their R&D expenditure to basic research, applied research, and development. The definitions closely follow the standard ones that were discussed in the introduction. As of 2016, the survey first breaks down R&D into research, defined as "the planned, systematic pursuit of new knowledge or understanding," and development, defined as "the systematic use of research and practical experience to produce new or significantly improved goods, services, or processes." Research is further divided into basic research, defined as "the activity aimed at acquiring new knowledge or understanding without specific immediate commercial application or use," and applied research, defined as "the activity aimed at solving a specific problem or meeting a specific commercial objective."

The main concern about the use of BRDIS/SIRD is probably internal validity. Specifically, firms might misinterpret the definitions of R&D composition. Also, we do not know how much effort firms invest in filling in the survey, which may result in noisy data. While these concerns are genuine, it should be emphasized that many of them do not threaten the validity of the main result of this paper. For instance, noise in the data, in itself, is not a likely explanation for what appears like a clear increase in the reported basic research expenditure during downturns. I think that the bigger issue is the gap between the high-quality data collected about large firms versus the more complicated situation, both in terms of sampling probability and in survey quality, collected from small ones. Specifically, a concern I have is that the acquisition of small firms by large firms during recessions results in large firms reporting basic research conducted in the small firm that was absent from the data before because of sample selection issues or lack of awareness among small firms of the proper way to categorize R&D. This alternative explanation cannot be addressed with the aggregate data and will require further work that is currently being conducted with Census microdata.

The survey itself, the sampling frame, and the aggregation methods have undergone substantial changes over the years. Overall, since the transition to BRDIS in 2008, the changes were quite minor and are not likely to have a substantial impact on the aggregates that will be discussed here. Further, the transition from SIRD to BRDIS between 2007-2008 had no impact on the sampling frame. They did result in a slight change in the questions related to R&D composition: where in SIRD firms were required to breakdown their R&D between basic research, applied research, and development, in BRDIS they were first required to break it down into research vs. development and only then to break down the research component to basic vs. applied research.

I discuss this change below and highlight why I do not think it is the driving force behind the changes we see in the great recession, let alone the general pattern of counter-cyclical investment in basic research. The sample size underwent substantial changes over the years.

The sampling frame that was described above has not changed much since it was first established in 1992. Before then, the design was different, with the survey being conducted at irregular intervals. For example, about 14,000 large firms with known R&D activity were surveyed in the 1987 survey. For instance, in the years 1987-1991, 1,700 of these firms kept on being surveyed on an annual basis, while the reports of other firms were imputed. A similar methodology was used in 1976-1981, and 1981-1986. Before that, the sampling frame was collected in even greater intervals. In addition, in older surveys, many of the firms were selected based on reports provided by US government agencies, and especially the Ministry of Defense, about their largest R&D contractors. In general, it seems that in these years the survey is characterized by more murky boundaries between private and public funding to R&D. Hence, I begin my sample in 1976.

#### 2.1.2 Other Data Sources

To get a proxy for publicly funded research, I am using data from the Higher Education Research and Development Survey (HERD) conducted by the NSF. HERD specifies the expenditure of universities and other research institutions on basic and applied research in the fields of science and engineering (S & E).<sup>2</sup>. To get a proxy for all public/academically funded research, I add the expenditures from HERD to federal funding to research conducted by the private sector, which appears in BRDIS.

Most of the relevant macro variables were taken from the Bureau of Economic Analysis. GDP was taken from the Real Gross Domestic Product series. All absolute sizes were converted to real terms using Gross Private Domestic Investment Implicit Price Deflator, where 2012=100. Interest rates are calculated according to the 1-Year Treasury Constant Maturity Rate series provided by the Board of Governors of the Federal Reserve System. The inflation rate was determined by

 $<sup>^{2}(1)</sup>$  Starting from 2008, HERD no longer distinguishes S & E from non-S & E research expenditure. However, as non-S & E amount to about 5% of total research spending, I disregard the differences. (2) HERD does not include data for 1978. I completed the series by using a linear extrapolation for this data point.

the percentage change in the series Consumer Price Index for All Urban Consumers: All Items, 1982-1984=100, from the Bureau of Labor Statistics.

### 2.2 Characterizing R&D in the Private Sector

When merging BRDIS/SIRD and HERD data, we discover that, as of 2015, private funding contributed to roughly 44% of total US research expenditures, with the federal government covering the majority of the remaining portion. Private funding also made up 26% of all basic research expenses. This suggests that a considerable segment of US research relies on private financing, which, in turn, could greatly influence innovation and, ultimately, economic growth.

To understand these ratios in absolute terms, (nominal) private basic research expenditure in 2015 amounted to \$16.3 Billion Dollars, applied research was \$44.3 Billion, and development expenditure equaled \$236 Billion. Historically, basic research constituted 19.5% of all research expenditure, and 4.7% of all privately funded R&D (namely, basic research, applied research, and development). Basic research constituted 0.5% of Private Non-Residential Fixed Investment (PNRI), while privately funded research amounted to 2.7% of it. All privately funded R&D activity equaled 11% of PNRI.

### 2.3 Private R&D Composition and the Business Cycle

To get some context for the coming results, let us briefly discuss the absolute size of privately funded basic research expenditure. When merging BRDIS/SIRD and HERD data, we discover that, as of 2015, private funding contributed to roughly 44% of total US research expenditures, with the federal government covering the majority of the remaining portion. Private funding also made up 26% of all basic research expenses. This suggests that a considerable segment of US research relies on private financing, which, in turn, could greatly influence innovation and, ultimately, economic growth.

The real value of private basic research expenditure in 2015 totaled \$16.3 billion, applied research reached \$44.3 billion, and development expenditures amounted to \$236 billion. At the time, the Private Non-Residential Fixed Investment (PNRI) was approximately \$2,450 billion. Historically, basic research accounted for 19.5% of all research expenditures and 4.7% of all privately funded R&D (including basic research, applied research, and development). Basic research made up 0.5% of PNRI, while privately funded research constituted 2.7% of it. Overall, privately funded R&D activity represented 11% of PNRI.

In the graphs appearing in Figure 1, we see that during the period 1986-2015, the growth of R&D expenditure of the private sector tends to move in the same direction as GDP, increasing in times of growth and declining in recessions. In contrast, the private sector's basic research expenditure seems to move in the opposite direction to GDP. Thus, the correlation coefficient between total R&D expenditure and GDP is  $\rho_{R&D,GDP} = 0.29$ , while that of basic research to GDP is  $\rho_{BR,GDP} = -0.35$ .

Further, basic research exhibits a dramatic increase in 2 out of 3 recessions that occur within this period. Thus, between 2007 to 2009 private basic research expenditure increased by more than a staggering 60%. Similarly, we witness a 55% increase between 1990 to 1991. In the year 2001, the private sector's investment in basic research increased by 19% - a high increase, but yet similar to its growth rate in the years 1999-2000. At the same time, during the years 1999 and 2000 total R&D expenditure was growing at a high rate of about 10%, while in 2001 it declined by 1%. So, here too we see basic research exhibiting strong growth in times when total R&D spending is declining.

To document the correlation between GDP and different types of research activities more formally, I set forth the following SVAR model:

$$y_t = A_1 y_{t-1} + A_2 y_{t+2} \dots A_j y_{t+j} + B\epsilon_t \tag{1}$$

Where  $y_t = [Y, RD]'$ , where Y is some aggregate variable (I picked GDP) and RD is a variable of R&D activity. Both variables are specified in percentage change. We assume that  $\epsilon$  is a set of orthogonal shocks:

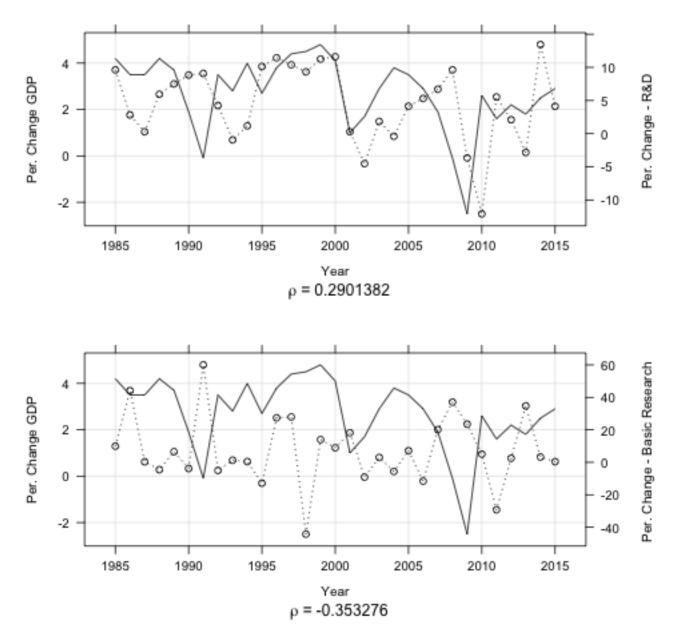
$$\epsilon_t = [\epsilon_{z,t}, \epsilon_{r,t}]' \sim N(0, \sigma_\epsilon), \qquad \sigma_\epsilon = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
(2)

The identifying assumption is that R&D expenditure has no contemporaneous impact on GDP, that is  $B_{1,2} = 0$ . This is because the process of transforming R&D into a product can take time. While this assumption might be more contentious when referring to development, it seems quite likely when discussing basic research, which typically requires much further work until it is manifested in new goods or improved production technology.

The object of interest is the contemporary impact of a GDP shock on R&D,  $B_{2,1}$ . I choose the number of lags by applying the Schwartz information criteria to all models with eight lags or less. In Table 1, I present the mean and standard deviation of  $B_{1,2}$  for different pairs of Y, R&D:

Variables	Value
Basic Research	-0.08 (0.037)**
Ratio Basic/Applied Research	-0.0124 (0.033)***
All R&D	$0.0009 \ (0.008)$

**Table 1:** SVAR results: coefficient on the contemporaneous impact of a GDP shock on various variables depicting private sector R&D expenditure



**Figure 1:** In both graphs, the solid line represents the percentage change of GDP. In the top graph, the dashed line represents the percentage change in total R&D expenditure of the private sector while in the bottom graph, it represents its total basic research expenditure. The data on R&D expenditure was taken from BRDIS/SIRD. Below each graph, we find the correlation coefficient between the two variables represented by the two lines, denoted  $\rho$ .

The table indicates that an unpredicted 1% decline in GDP is associated with an 8% increase in the private sector's basic research expenditure. The result is statistically significant at a 5% level. This decline is also associated with an increase of 12.5% in the ratio between the growth of basic research vs. applied research expenditure. Lastly, we see no statistically significant correlation between GDP growth and R&D expenditure.

Finally, a regression model was run to test whether weak demand may explain the countercyclical behavior of basic research expenditure. For that purpose, I approximate the demand using consumption, and I use the model:

$$\Delta\% \text{Private BR} = \alpha + \beta_1 \Delta\% \text{Consumption} + \beta_2 \Delta\% \text{Public BR} + \beta_3 \Delta\% \text{Interest} + \beta_4 \Delta\% \text{Inflation}$$
(3)

Where  $\Delta\%$  denotes percentage change compared to the previous year, Private BR denotes the total basic research expenditure in the private sector, and Public BR denotes the total basic research expenditure in the public sector, including federal funding and spending of higher education institutions.

Dependent Variable:	$\Delta$ %Private BR Expenditure			
Model:	(1)	(2)	(3)	(4)
$\Delta$ % Consumption	-4.015*	-4.716*	-4.527*	-4.971**
	(2.042)	(2.337)	(2.305)	(2.331)
$\Delta$ % Public BR Exp.		0.6436	0.5072	0.4192
		(1.006)	(0.9953)	(0.9946)
$\Delta$ % Interest			-0.0252	-0.0286
			(0.0186)	(0.0188)
$\Delta$ % Inflation				3.375
				(3.077)
Fit statistics				
Observations	29	29	29	29
$\mathbb{R}^2$	0.12525	0.13881	0.19758	0.23589
Adjusted $\mathbb{R}^2$	0.09285	0.07256	0.10129	0.10854

*IID standard-errors in parentheses* 

Signif. Codes: \*\*\*: 0.01, \*\*: 0.05, \*: 0.1

**Table 2:** Regression of percentage change of private expenditure on basic research on the percentage change in consumption, used as a proxy for demand, 1986-2016. I also included controls for percentage change in public basic research expenditure, interest, and inflation

The results appear in Table 2. When controlling for these other factors, we see a negative

correlation between demand (consumption) and private investment in basic research - consistent with a Schumpeterian growth theory. Most importantly, note that public investment in basic research has no significant correlation with private investment. Further, comparing column 1 and column 2, we find that including it in the analysis has a negligible impact on R-squared, an insignificant ability to explain changes in basic research over time. This is evidence against the claim that the counter-cyclical behavior of basic research originates from higher government subsidies to this type of activity during a recession that was misreported as firm spending. If that had been the case, we would have expected that a rise in the reported level of basic research among firms would be manifested in the government reporting greater spending on facilitating such activity, which is not the case. Also, we see that when controlling for the interest rate and inflation. According to these regressions, a 1% decline in consumption implies a 4% increase in basic research expenditure.

# 3 Model

#### 3.1 Settings

The economy exists for an infinite number of periods, t = 0, 1, 2... It is populated by a continuum of size L(t) of households, a single final good producer, a continuum of upstream firms, a continuum of downstream firms, and a continuum of R&D firms.

#### 3.1.1 Households

We assume that the measure of households, L(t), is growing by some exogenous rate,  $g_l$ , so that  $L(t+1) = g_l L(t)$ . Households are infinitely lived and maximize their discounted expected CRRA utility function:

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t \frac{C_t^{1-\sigma}-1}{1-\sigma}$$

Choose how much to consume c, and how much to save/borrow, a.

Each household is endowed with 1 unit of manual labor and  $\gamma$  units of R&D labor. Also, Households have access to a borrowing/lending technology with a rate of return of  $1 + \bar{r}$ . In each period, they will choose how much to work, how much to consume, and how much to save. Lastly, households own the firms and attain dividends from their profits.

#### 3.1.2 Downstream Producer

The final-good producer will produce the single consumption good, C, using intermediate goods sold by upstream firms. The final good's producer production function is given by:

$$Y_t = exp(Z)exp(\int_0^1 ln(y_{i,t})$$
(4)

Where Z is an exogenous productivity shock. We shall assume that Z evolves according to:

$$Z' = \rho_z Z + \epsilon, \quad \rho_z \in (0, 1), \epsilon \sim N(0, \sigma_\epsilon),$$
(5)

The final good producer acts as a price taker for the price of the final good, P. Given the prices suggested by each upstream firm for good j, he chooses which firm to buy from and what quantity,  $y_j$  to purchase. We will assume, WLG, that if the downstream producer is indifferent between buying from a few different firms he randomly picks one of them and buys from it.

#### 3.1.3 Upstream Firms

There is a continuum of size 1 of upstream firms that produce intermediary goods. Firms maximize their stream of expected profits while discounting future income by a rate of 1 + r. Each firm, *i*, can produce good *j* according to the production technology:

$$y_{i,j} = (q_{i,j}l_{i,j})^{\alpha} (k_{i,j})^{1-\alpha}$$

Where  $l_{i,j}, k_{i,j}$  denote the labor and capital (correspondingly) that the firm assigns to the production of j, while  $q_{i,j}$  is an idiosyncratic variable representing firm i technology for creating j. Each upstream firm will be characterized by a vector of measure 1 of quality levels,  $\{q_{i,j}\}_{i \in [0,1]}$ .

Upstream firms will be able to augment their production technology by purchasing patents exclusive rights of use in state-of-the-art technology, that is  $q_{i,j^*} > \max_{j \in J} \{q_{i,j}\}$ .

In each period, each upstream firm chooses how much labor and how much capital to employ in each one of its production lines. The firms will take the wage for manual labor,  $w_m$ , and the rent on capital, r, as given. Also, each firm will decide which price to post for each intermediary good,  $p_{i,j}$ . We assume that firms simultaneously post prices and that the choice of price for each firm and for each intermediary good is optimal given the prices posted by other incumbents.

#### 3.1.4 R&D Firms

The economy is populated by a continuum of size 1 of R&D firms. Each period, each of these firms can make an attempt to create a new blueprint by employing R&D labor in applied research. The firm can also employ R&D labor for *basic research*, which augments its knowledge. The knowledge it acquires will facilitate better prospects of new innovation in future periods.

The process of innovation in the economy follows a few steps. In each period, all R&D firms are organized according to some random order. Firms are sequentially "called" to try to innovate. When a firm *i* is called, it is randomly assigned to a variety *j* that did not witness innovation by another firm that was called before it. The firm attempts to innovate by employing applied research. If the firm innovates, it creates a new blueprint. Let  $q_{j,-}^*$  denote the state-of-the-art technology for creating variety *j* at the beginning of the period before any innovation took place. If the firm creates a blueprint, the blueprint allows creating good *j* with a technology level of:

$$q_i^* = \bar{\lambda} q_{k,-}^*, \quad \bar{\lambda} > 1 \tag{6}$$

That is, innovation implies a constant "jump" increase in the quality of the production technology. The fixing of the changes in the quality of the production technology allows us to focus on a different margin - the probability of success.

Specifically, assume that an R&D firm probability of successfully innovating is given by:

$$F(U, n, r_a) = \min\left\{r_a^{\omega} \frac{n^{\theta}}{U^{\omega+\theta}}, 1\right\}, \quad \omega, \theta \in (0, 1)$$
(7)

Where  $r_a$  denotes the units of applied research that were utilized, n is the firm's knowledge. U is aggregate knowledge utilization, which we defined as the total development efforts conducted by all R&D firms throughout all time:

$$U_{\tau} = \underline{U} + \sum_{t=0}^{\tau} \int_{j \in J} \eta r_{a,j,t} dj, \quad \underline{U} \in \mathbb{R}$$
(8)

F() is increasing in  $r_a$ , as more development efforts yield greater rates of innovation, and concave in  $r_a$ , to reflect decreasing marginal utility in R&D. F() is increasing in n as the more knowledge the firm acquired, the higher the likelihood of coming up with new technologies. Lastly, F() is decreasing in U. The intuition behind it is that the more research already conducted in the past by other incumbents, the harder it is to come up with a novel finding. One can think of this setting as inspired by the literature that regards R&D as a process of *search* in the spirit of Bental and Peled (1996).

Alongside, the firm knowledge accumulation is given by:

$$n' = \rho_n n + \delta r_b^{\kappa} n^{1-\kappa} + \delta_e R_b + \delta_a r_{b,a}^{\psi}, \quad \delta \in \mathbb{R}_+, \rho \in (0,1)$$
(9)

Where  $r_b$  is a measure of basic research conducted by the firm,  $R_b = \int_{j \in J} r_{b,j} dj$  is the total measure of basic research conducted by all incumbent firms, and  $r_{b,a}$  is the measure of basic research conducted in the academia or by the government. That is, the equation embeds the assumption of a knowledge spillover from basic research conducted by one firm on other incumbents. It also incorporates a spillover from research conducted in academia, embedded in the variable  $r_{b,a}$ . The magnitudes of the knowledge spillover, when compared to the importance of research conducted in-house, is reflected in the sizes of the coefficients  $\delta, \delta_e, \delta_a$ . Lastly, I assume that knowledge persists over time, but that persistence is imperfect and some may get lost. The loss of knowledge is embedded in the assumption  $\rho_n$ , which I shall regard as the knowledge persistence parameters, being smaller than 1.

Thus, it is plain to see that in our settings R&D is determined by two types of externalities. When other incumbents develop new knowledge, through  $r_b$ , they create a spillover which makes it easier for the firm to innovate. On the other hand, when other incumbents exploit existing knowledge and embed it into new blueprints through applied research, they deplete the pool of potential novelties and make it harder for the firm to come up with a new blueprint.

#### 3.1.5 Market for Patents

Patents in our economy will be sold in a competitive setting for a price  $\nu()$ . A patent will grant its holder exclusive rights for state-of-the-art technology for creating a specific intermediary good, j. That is, for the patent holder, i, it will be the case that:  $q_{i,j^*} > \max_{j \in J} \{q_{i,j}\}$ . We will assume that once a patent for a good j is granted, all previous patents for the same good are nullified so that all other incumbents can now produce j at the second-best technology available, that is by  $q_j^*/\bar{\lambda}$ . The assumption, taken from Barlevy (2007), leaves the model tractable and avoids questions of strategic behavior by firms but the patent holder itself.

### 3.2 Defining Equilibrium

Let  $\overrightarrow{S} = (Z, L, \overline{Q}, U, N, K, A)$  Where  $\overline{Q} := exp(\int_0^1 ln(q_i)di, K$  denotes aggregate capital, and A is aggregate household savings. A candidate for a recursive competitive equilibrium will consist of:

- 1. A value function for the firm  $V(): \mathbb{R}^8 \to \mathbb{R}$ , and R&D policy functions  $r_a(), r_b(): \mathbb{R}^8 \to \mathbb{R}_+$ .
- 2. A pricing, production, labor demand, and profit functions for a monopolist:  $y_i(), p_i(), k_i() : \mathbb{R}^8 \to \mathbb{R}, l_i(), \pi() : \mathbb{R}^7 \to \mathbb{R}_+$
- 3. Household value function:  $V^h() : \mathbb{R}^8 \to \mathbb{R}$ , and corresponding policy functions  $A() : \mathbb{R}^8 \to \mathbb{R}$ ,  $C() : \mathbb{R}^5 \to \mathbb{R}_+$ .
- 4. Price of the final good, quantity produced, and total expenditure for the final good producer,  $P(), Y(), E() : \mathbb{R}^7 \to \mathbb{R}_+; \text{ A demand function for each intermediate good, } y_i^d() : \mathbb{R}^8 \to \mathbb{R}_+.$
- 5. A quality mapping:  $Q(): \mathbb{R}_7 X[0,1] \to \mathbb{R}_+$ .
- 6. A transition rule:  $H(): \mathbb{R}^7 \to \mathbb{R}^4$ .

The candidate will constitute an equilibrium if the following holds:

- 1.  $V(\vec{S}, n)$  solves the firm problem, with  $r_a(\vec{S}, n), r_b(\vec{S}, n)$  being the corresponding policy functions.
- 2.  $V_h(\overrightarrow{S}, a)$  solves the HH problem, with  $C(\overrightarrow{S}, a), a(\overrightarrow{S}, a)$  being the corresponding policy functions.
- 3. The monopolist is optimizing:  $p_i(\overrightarrow{S}, q^*), y_i(\overrightarrow{S}, q^*), l_i(\overrightarrow{S}, q^*), k_i(\overrightarrow{S}, q^*)$  satisfy (2);  $\pi(\overrightarrow{S})$  satisfies (3).
- 4. The downstream producer is optimizing:  $y_i^d(\overrightarrow{S}, q^*)$  satisfies (1); Makes a zero profit:

$$P(\overrightarrow{S}) * Y(\overrightarrow{S}) = E(\overrightarrow{S})$$

Feasibility:

5. The downstream producer expenditure is given by:

$$E(\overrightarrow{S}) = \int_0^1 y_i^d(\overrightarrow{S}, Q(\overrightarrow{S}, i)) p_i(\overrightarrow{S}, Q(\overrightarrow{S}, i)) di$$

- 6. Feasibility of monopolist's choice:  $(l_i(\overrightarrow{S})Q(\overrightarrow{S},i))^{\alpha}(k_i(\overrightarrow{S},Q(\overrightarrow{S},i))^{1-\alpha} = y_i(\overrightarrow{S},Q(\overrightarrow{S},i))$
- 7. Feasibility of the downstream producer's choice:

$$Y(\overrightarrow{S}) = Zexp(\int_0^1 ln(y_i^d(\overrightarrow{S}, p_i(\overrightarrow{S}, Q(\overrightarrow{S}, i))di)$$

8. For a measure  $[r_a(\overrightarrow{S}, n))^{\omega} \frac{n^{\theta}}{U^{\omega+\theta}}]$  of goods it is the case that:

$$Q(H(\overrightarrow{S}),i) = \overline{\lambda}Q(\overrightarrow{S},i)$$

For the rest:

$$Q(H(\overrightarrow{S}),i) = Q(\overrightarrow{S},i)$$

Consistency:

9. Consistency/representative agent:

$$\begin{aligned} H_1(\overrightarrow{S}) &= Q' = \int_0^1 Q(H(\overrightarrow{S}), i) di = \\ &\int_0^{(r_a(\overrightarrow{S}, K))^\omega} \frac{K^\theta}{U^{\omega+\theta}} \,\overline{\lambda} Q(\overrightarrow{S}, i) di + \int_{(r_a(\overrightarrow{S}, K))^\omega}^1 \frac{K^\theta}{U^{\omega+\theta}} \,Q(\overrightarrow{S}, i) di \\ H_2(\overrightarrow{S}) &= N' = \rho N + r_b(\overrightarrow{S}, N) \\ &H_3(\overrightarrow{S}) = A' = a(\overrightarrow{S}, A) \\ &H_4(\overrightarrow{S}) = L' = \Lambda(L) \end{aligned}$$

10. Representative agent:

$$N = n$$

$$A = a$$

$$K(\overrightarrow{S}) = \int_0^1 k_i(\overrightarrow{S}, Q(\overrightarrow{S}, i)) di$$

Market Clearing:

10. Both labor markets clear:

$$r_{a}(\overrightarrow{S},k) + r_{b}(\overrightarrow{S},k) = \gamma L$$
$$\int_{0}^{1} l_{i}(\overrightarrow{S},Q(\overrightarrow{S},i))di = L$$

11. The output market clears:

$$Y(\overrightarrow{S}) = LC(\overrightarrow{S}, a)$$

12. The market for each intermediate good clears:

$$y_i^d(\overrightarrow{S}, p_i(\overrightarrow{S}, Q(\overrightarrow{S}, i))) = y_i(\overrightarrow{S}, Q(\overrightarrow{S}, i))$$

# 4 Solving the Model

Due to the Cobb-Douglas production function of the downstream producer, he will have the same expenditure, which we will denote by E, on purchasing each one of the intermediary goods. Trivially, the downstream producer will buy only at the cheapest price available, which we shall denote  $p_j^*$ . Its demand for good j will be given by:

$$y_j^*(p_j) = \frac{E}{p_j} \tag{10}$$

If a few firms post the cheapest price, we will assume, WLG, will randomly choose one to buy from.

Now, let us look at the optimization problem of a firm that holds a patent for state-of-the-art technology to produce good j. It maximizes:

$$\pi(E, W_m, r, q_i^*, \tilde{q}_i) = \max_{\{p_i, y_i, l_i, k_i\}} \{ (p_i y_i - W_m l_i - rk_i) \mathbb{I} \{ p_i \le \min_{j \in J} \{ p_{i,j} \} \}$$
(11)  
s.t :  $y_i = \frac{E}{p_i}, \quad y_i = (q_i^* l_i)^{\alpha} k_i^{1-\alpha}$ 

Recall that firms have constant returns to scale production function, and let  $\tilde{c}_j$  denote the production cost with the second best technology,  $\tilde{q}_j$ . For any price that is weakly higher than that, the patent holder wins the market and hence has a revenue of E. He will maximize profits by picking the price in which he is required to produce the minimal quantity to attain this revenue or the highest price he can charge without losing business, that is  $p_j^* = \tilde{c}_j$ .

In the appendix, I show that a firm with productivity  $q_j$  can produce at a minimal marginal cost of:

$$c_j(q_j) = r \frac{\left(\frac{1-\alpha}{\alpha} \frac{w}{r}\right)^{\alpha}}{q^{\alpha}} + W \frac{\left(\frac{1-\alpha}{\alpha} \frac{w}{r}\right)^{\alpha-1}}{q^{\alpha}}$$

Thus, the production cost of the monopolist will allow it to produce at a cost that is cheaper by  $\lambda^{\alpha}$  compared to the second best competitor, or  $p_i = \tilde{c}_i = \lambda^{\alpha} c_i^*$ . Denoting its production cost by  $c^*$ , we find that its profit will be given by:

$$\pi = y_i[p_i - c^*] = \frac{E}{p_i}[p_i - c^*] = E[1 - \frac{c^*}{p_i}] = E[1 - \lambda^{-\alpha}]$$

Or, more formally:

$$\pi(E, W_m, r, q_j^*, \tilde{q}_j) = (1 - \bar{\lambda}^{-\alpha})E$$
(12)

Note that the profit of the patent holder is independent of the quality level of the intermediary good it produces. Since patents of each good produce the same profit our assumption that firms randomly choose which good to innovate upon seems benign. Further, since the downstream producer breaks even, its expenditure exactly equals the households' expenditure on consumption. Hence, we find that firms' profits are proportional to aggregate consumption spending, as they always constitute exactly  $(1 - \lambda^{-1})$  out of it.

At this phase, we are able to pin down the value of a patent when it is leased for sale. Recall, the patent is first offered for sale after R&D firms attempted innovation but before any production took place. Hence, the patent will generate profit for its owner at least in the current period. As for future periods, the patent might be infringed. In each period, F() of randomly selected patents are infringed. Therefore, the probability that a patent is infringed is F(). Hence, the value of a patent is:

$$\nu_t = \pi_t + \mathbb{E} \sum_{\tau=1}^{\infty} \frac{\prod_{t=1}^{t+\tau} (1 - F_{t+\tau})}{(1+\tau)^{\tau}} \pi_{t+\tau}$$
(13)

Due to our assumptions, that will also be the price at which the patent will be sold.

Now, we can proceed to study the behavior of the R&D firms. Denote the average production quality by  $\overline{Q} = \int_0^1 q_j dj$ , and let  $\overrightarrow{S} = (Z, L, K, \overline{Q}, U, N)$  be a vector of all aggregate state variables. We can write down the R&D firm's problem as:

$$V(\overrightarrow{S},n) = \max_{r_a,r_b} \{ r_a^{\omega} \frac{n^{\theta}}{U^{\omega+\theta}} \nu(\overrightarrow{S}) - W_r(\overrightarrow{S})(r_a+r_b) + \frac{1}{1+r} \mathbb{E}[V(\overrightarrow{S}',n')] \}$$
  
s.t  $n' = \rho n + \delta r_b^{\kappa} n^{1-\kappa},$   
 $\overrightarrow{S}' = H(\overrightarrow{S})$  (14)

Define the Lagrangian for the firm's problem:

$$L = r_a^{\omega} \frac{n^{\theta}}{U(\overrightarrow{S})^{\omega+\theta}} \nu(\overrightarrow{S}) - W_r(\overrightarrow{S})(r_a + r_b) - \mu(n' - \rho n - \delta r_b) + \frac{1}{1 + \overline{r}} \mathbb{E}V(S', n')$$

Taking FOC we get:

$$\frac{\partial L}{\partial r_a} : \omega r_a^{\omega - 1} \frac{n^{\theta}}{U^{\omega + \theta}} * \nu(\overrightarrow{S}) = W_r(\overrightarrow{S})$$

$$\frac{\partial L}{\partial r_b}: \delta \mu = W_r(\overrightarrow{S})$$

$$\frac{\partial L}{\partial n'}: \mu = \frac{1}{1+\bar{r}} \mathbb{E} \frac{\partial \nu(S',n')}{\partial n'}$$

Using ENT:

$$\frac{\partial V(S,n)}{\partial n} = \theta r_a^{\omega} \frac{n^{\theta-1}}{U^{\omega+\theta}} V(\overrightarrow{S}) + \rho \mu$$

Combining the last 3 equations and rearranging we get Euler's equation:

$$\omega r_a^{\omega-1} \frac{n^{\theta}}{U^{\omega+\theta}} * \nu(\overrightarrow{S}) = \frac{1}{1+\overline{r}} \mathbb{E}[r_a^{'\omega} \frac{n^{'\theta}}{U^{'\omega+\theta}} \nu(\overrightarrow{S}') * (\frac{\delta\theta}{n'} + \frac{\rho\omega}{r_a'})]$$

On the LHS of the equation, we find the marginal return on conducting more applied research, which may result in selling an additional patent for  $\nu(\vec{S})$ . On the RHS, we have the expected return from additional basic research.

# 4.1 Balanced Growth Path

Implicitly define the variable  $\phi$  as the positive number satisfying:

$$\phi = \left(\frac{\kappa\theta\delta^{1/\kappa}}{\omega(\frac{1+\bar{1}}{\lambda\phi} - \rho)(g_l - \rho)^{\frac{1}{\kappa} - 1}}\right)^{\theta} \left(\frac{g_l - 1}{\eta}\right)^{\omega + \theta}$$
(15)

And define the following conditions:

C1. (consumption grows at a constant rate):

$$\beta(1+r^*)\frac{\lambda^{\phi(1-\sigma)}}{1+\phi(\lambda-1)} = 1$$

C2. (The expected value of a patent is finite):

$$1 + r^* > (1 - \phi)\lambda^{\phi}g_l$$

C3. ( $\phi$  is a probability measure)

$$0 < \phi < 1$$

**Theorem 4.1.** If conditions 4.1 hold, then given the initial population size,  $L_0$ , the economy has a unique BGP on which:  $r_a, r_b, n, U$  grows at a rate of  $g_l$ , while,  $Y, \pi$  grows at a rate of  $g_l \lambda^F$ ,  $\bar{Q} = exp(\int_0^1 ln(q_i^*))$ , grows at a rate of  $\lambda^{\phi}$ , while F is constant and equals  $\phi$  (defined in 15.

*Proof.* Labor demand is inelastic and thus:

$$r'_a = g_l r_a, r'_b = g_l r_b \tag{16}$$

Note that  $r'_a = g_l r_a$  immediately implies that  $U' = g_l U$ , and thus:

$$g_l U = U' = U + \eta r_a \tag{17}$$

The constant growth of the economy implies that patenting, F, is constant. Recall that by definition:

$$F = r_a^{\omega} \frac{n^{\theta}}{U^{\theta + \omega}} \tag{18}$$

Since it is constant and  $r_a$ , U increases at a rate of  $g_l$ , it must be the case that n also increases at a rate of  $g_l$ , and hence:

$$g_l n = n' = \rho_n n + \delta r_b^{\kappa} n^{\kappa} \tag{19}$$

Now, we can rewrite the Euler equation as:

$$\omega r_a^{-1} F * \nu(\overrightarrow{S}) = \frac{1}{1 + \overline{r}} \mathbb{E}[F'\nu(\overrightarrow{S}') * (\frac{\delta\theta}{n'} + \frac{\rho\omega}{r'_a})]$$

Bear in mind that F = F'. Also, we know that  $\nu(\vec{S})$  grows at a rate of  $g_l \lambda^F$ . Using this, and plugging in eq. 19, we get:

$$r_a = \frac{\omega(\frac{1+\bar{1}}{\lambda^{F^*}} - \rho)(g_l - \rho)^{\frac{1}{\kappa} - 1}}{\kappa \theta \delta^{1/\kappa}} n \tag{20}$$

Lastly, we know that:

$$r_a + r_b = \gamma L \tag{21}$$

And L is uniquely pinned down for each period by  $g_l, L_0$ . Hence, we have 5 equations, 17, 18, 19, 20, 21, in 5 unknowns:  $U, n, r_a, r_b, F$  - which admits only a single solution.

Define:

$$C_1 = \frac{\omega(\frac{1+\bar{r}}{\lambda^{F^*}} - \rho)}{\delta\theta}$$
$$C_2 = \frac{\bar{g}_l - \rho}{\delta}$$

Using the five equations just stated, we can show that:

$$n^* = \frac{\gamma L}{C_1 + C_2}$$

$$r_a^* = \frac{C_1 \gamma L}{C_1 + C_2}$$

$$r_b^* = \frac{C_2 \gamma L}{C_1 + C_2}$$

$$U^* = \frac{\eta C_1 \gamma L}{(\bar{g}_l - 1)(C_1 + C_2)}$$

$$F^* := \left(\frac{\kappa \theta \delta^{1/\kappa}}{\omega(\frac{1 + \bar{1}}{\lambda^{F^*}} - \rho)(g_l - \rho)^{\frac{1}{\kappa} - 1}}\right)^{\theta} \left(\frac{g_l - 1}{\eta}\right)^{\omega + \theta}$$

By construction, these equations satisfy the optimality of R&D firms' actions given that  $r_a, r_b.U, n$ grow at a rate of  $g_l$ . On this BGP, the value of a patent is the expected value of the future profits from holding it:

$$\nu(S_t) = \Sigma_{t=\tau}^{\infty} \frac{(1-\phi)^{\tau}}{(1+r)^{\tau}} \pi(S_t + \tau) = \Sigma_{t=\tau}^{\infty} \frac{(1-\phi)^{\tau}}{(1+r)^{\tau}} \pi(S_t) (\lambda^{\phi} g_l)^{\tau} = \frac{\pi(S_t)}{1 - \frac{(1-\phi)\lambda^{\phi} g_l}{1+r}}$$
(22)

The first equality is derived from the fact that firms' profits are always a constant share of total consumption, and on the BGP, consumption is a steady share of GDP and hence rises at a rate of  $g_l \lambda^{\phi}$ . The second equality implies that the value of a patent is well-defined if and only if:

$$\frac{(1-\phi)\lambda^{\phi}g_l}{1+r}\pi(S_t) < 1$$

Which is satisfied by assumption C2 in 4.1. Now, for consumption to constitute a constant share of GDP, it should be optimal for households to keep their consumption-to-savings ratio constant as their income increases. This is assured by condition (1), and the full details appear in the appendix.

Now, we examine the behavior of  $\overline{Q}$ . Let us write down the behavior of  $\overline{Q}$  in the next period, given that a share F of all technologies jumps by  $\lambda$ , while the rest remain the same:

$$\bar{Q}' = exp(\int_{0}^{1} ln(q_{i}'^{*})di) = exp(\int_{0}^{1-F} ln(q_{i}^{*})di + \int_{1-F}^{1} ln(q_{i}^{*}\lambda)di) = exp(\int_{0}^{1} ln(q_{i}^{*})di + Fln(\lambda)) = exp(\int_{0}^{1} ln(q_{i}^{*})di + ln(\lambda^{F})) = \lambda^{F}\bar{Q} \quad (23)$$

Note that on the BGP, the inputs of innovation,  $r_a, r_b$ , are growing exponentially, while the growth generated from R&D,  $\lambda^{\phi}$ , remains constant. That is, we witness a continuing decline in R&D productivity. Yet, despite this decline in productivity, the share of output allocated to R&D remains constant over time. This is because firms are compensated for the lower productivity in higher value by holding a patent due to the economy's overall growth.

This model feature is consistent with evidence suggesting a long-term trend of increasing the

difficulty of innovation, documented by Kortum (1997) and Bloom et al. (2020). The success of this model in capturing this long-term trend is important. The same forces that drive it can also play a role in shaping the response of technological development to shocks. Indeed, as we shall see below, they do.



Figure 2

# 5 Calibration

### 5.1 Stationary Model

In order to calibrate the model, I write an analogous stationary model that can be more easily solved using standard techniques. Define  $\tilde{x} = x/L$ , normalize  $Q = \int_0^1 ln(q_i)di = 1$  and let  $\tilde{S} = (Z, 1, 1, \tilde{U}, \tilde{N})$ . Also, assume a stochastic discount factor of  $g_l \lambda^{F'}$ .

Output is given by the equations:

$$\tilde{Y} = e^{Z+1} \tag{24}$$

$$Z' = \zeta Z + \epsilon, \ \epsilon N(0, \sigma_{\epsilon}) \tag{25}$$

R&D labor supply is constant and equals to Gamma:

$$\tilde{r}_a + \tilde{r}_b = \gamma \tag{26}$$

The representative R&D firm's problem is:

$$V(\tilde{S},\tilde{n}) = \max_{\tilde{r}_a,\tilde{r}_b} \{ \tilde{r}_a^{\omega} \frac{\tilde{n}^{\theta}}{\tilde{U}^{\omega+\theta}} \nu(\tilde{S}) - W_r(\tilde{S})(\tilde{r}_a + \tilde{r}_b) + \mathbb{E}[\frac{g_l \lambda^{F(S')}}{1+r} V(\tilde{S}',\tilde{n}')] \}$$
(27)

Where:

$$g_l \tilde{n}' = \rho \tilde{n} + \delta \tilde{r}_b^{\kappa} \tilde{n}^{1-\kappa}$$

$$g_l \tilde{U}' = \tilde{U} + \int_{j \in J} \eta \tilde{r}_{a,j} dj$$
(28)

And where F is defined by:

$$\tilde{F} = \tilde{r}_a^{\omega} \frac{\tilde{n}^{\theta}}{\tilde{U}^{\omega+\theta}} \tag{29}$$

Taking FOCs and applying the ENT we can derive the Euler equation for the standardized model:

$$\omega \tilde{r}_a^{-1} \tilde{F} \nu(\tilde{S}) = \delta \kappa(\frac{\tilde{n}}{\tilde{r}_b})^{1-\kappa} \mathbb{E} \Big[ \frac{\lambda^{\tilde{F}'}}{1+r} \tilde{F}' \nu(\tilde{S}') \Big( \frac{\theta}{\tilde{n}'} + \frac{\rho \omega + \delta(1-\kappa) \big(\frac{\tilde{r}_b'}{\tilde{n}'\kappa}\big)}{\tilde{r}_a' \delta \kappa \big(\frac{\tilde{r}_b'}{\tilde{n}_{t+1}}\big)^{\kappa-1}} \Big) \Big]$$
(30)

Lastly, note that the value of a patent is as follows.

$$\nu(\tilde{S}) = \pi(\tilde{S}) + \mathbb{E}\Big[\frac{\lambda^{F(\tilde{S}')}g_l}{1+r}(1 - F(\tilde{S}')\pi(\tilde{S}') + \frac{\lambda^{F(\tilde{S}') + F(\tilde{S}'')}g_l^2}{(1+r)^2}(1 - F(\tilde{S}')(1 - F(\tilde{S}'')\pi(\tilde{S}'') + \dots\Big]$$
(31)

This can also be written recursively as:

$$\nu_t - \pi_t = \mathbb{E}\Big[\frac{g_l \lambda^{F_{t+1}} (1 - F_{t+1})}{1 + \bar{r}} \nu_{t+1}\Big]$$
(32)

In the appendix, I show that the R&D company policy in the stationary model,  $\tilde{r}_a^*, \tilde{r}_b^*$ , implies the policy in the original model as  $r_a = \tilde{r}_a^* L, r_b = \tilde{r}_b^*$ . The standardized model will be used for the calibration.

### 5.2 Calibrating the model

To calibrate the model, we use a set of aggregate moments from the period 1990-2008. These moments include the labor share, the hazard rate of patents, the TFP growth rate, the GDP growth rate, the share of the labor force in R&D, and the ratios of basic/applied research and R&D wage to non-R&D wage.

The labor share is 0.6%, which we use to determine the value of  $\alpha$  as 0.6. The hazard rate of patents, that is the probability that a patent will be infringed during a given year, is estimated to be approximately 0.1 by Kortum (1997). Hence, we can set  $F^* = \phi = 0.1$ .

To map the TFP moments to the model, we use the relation  $TFP = e^Z * e^Q$ , which implies that:

$$log(TFP) = Z + Q$$

Q is growing at a constant rate of  $\lambda^{\alpha F}$ , so we can write:

$$log(TFP) = Z + \lambda^{\alpha\phi}t$$

Thus, I run the regression:

$$log(TFP) = \mu t + \xi_t \tag{33}$$

The coefficient 0.9, which we use to determine the value of  $\lambda^{\phi\alpha}$  as 1.009 and the value of  $\lambda$  as 1.16. The GDP growth rate is 2.85, which we use to determine the value of  $g_l \lambda^{\phi}$  as 1.0285 and the value of  $g_l$  as 1.013.

Now, I apply a regression model to the residuals from regression 33. The model is:

$$\xi_t = \rho_z \xi_{t-1} + \epsilon \tag{34}$$

I find  $\rho_z = 0.5$  and  $\sigma_{\epsilon} = 0.6$ .

The share of the labor force in R&D is 0.4, as reported by the Bureau of Labor Statistics (BLS), which we use to determine the value of  $\gamma$  as 0.004. From the macro literature, we use values of  $\beta$  as 0.96 and  $\bar{r}$  as 1.04.

We attain the ratio of basic/applied research expenditure in the private sector from BRDIS/SIRD data. It is about 23%. Thus:

$$\frac{r_a}{r_b} = \frac{C_2}{C_1} = 0.23$$

The ratio of R&D wage to non-R&D wage is determined using the average full-time equivalent

compensation for R&D employees from the NSF Survey of Research and Development (SIRD) and the average FTE compensation in the US, giving a ratio of 3.12.

In the model, the ratio is:

$$\frac{W_r^*}{W_m^*} = \frac{\omega \frac{C_1 + C_2}{\gamma C_1} F^*(\lambda - 1)}{1 - \frac{(1 - F^*)g_l \lambda^{F^*}}{1 + \bar{r}}} = 3.12$$

This allows us to infer that  $\omega = 0.57$ .

To model the average time between the publication of a scientific paper and its appearance in a patent citation, we assume that the firm randomly picks "knowledge" (a scientific paper) from all the knowledge it accumulated in the past. We set  $\rho$  to be such that the mean of the draw would be knowledge from 9 years ago. Using a sequence of weights, we find  $\rho_n = 0.92$ .

To simplify the computation, we define  $\phi$  as a parameter that represents the steady-state value of F. We use this parameter to derive  $\eta$  that generates it given the values of the other parameters of the model. This allows us to ensure that the steady-state value of F is between 0 and 1 and makes computation easier.

$$\eta = \frac{C1^{-\frac{\theta}{\theta+\omega}}(g_l-1)}{\phi^{\frac{1}{\theta+\omega}}}$$

Labor Share	0.6	BEA.
Patents' hazard rate	0.1	Kortum. (1997)
TFP growth	0.9%	BEA.
GDP growth	2.85%	BEA.
Share of R&D GDP.	0.015	BEA.
Real Interest Rate	1.04	Standard.
Average time until citing a scientific paper	9 years	Marx (2019).
Persistence of TFP shocks $(AR(1))$	0.5	BEA
Variance of TFP innovations $(AR(1))$ .	0.6	BEA
Basic/applied research	0.23	SIRD (NSF).
Average R&D wage/average wage	3.12	SIRD (NSF).

Table 3: Target Moments, 1990-2008

Now, we are left with estimating  $\delta, \kappa, \theta$ , and  $\eta$ . For the time being, we will randomly pick:  $\kappa = 0.9, \delta = 0.3$ , which gives us: C2 = 0.2, and hence C1 = 0.86. Plugging this to our equation for C1 will provide us with  $\theta$ :

$$C1 = (\frac{(\frac{1+r}{\lambda^F} - \rho)}{\delta \kappa (\frac{g_l - \rho}{\delta})^{(\kappa - 1)/\kappa} \theta} - \frac{(1 - \kappa)(\frac{g_l - \rho}{\delta})^{1/\kappa}}{\kappa \theta}) \omega$$

Thus:

$$\theta = \left(\frac{\left(\frac{1+r}{\lambda^F} - \rho\right)}{\delta\kappa\left(\frac{g_l - \rho}{\delta}\right)^{(\kappa-1)/\kappa}} - \frac{(1-\kappa)\left(\frac{g_l - \rho}{\delta}\right)^{1/\kappa}}{\kappa}\right) * \frac{\omega}{C1}$$

Which gives us  $\theta = 0.4$ . Now, we plug this into our equation for the patenting rate on the BGP, that is eq. 15, to get:

$$\eta = \frac{(g_l - 1)C1^{-\theta/(\theta + \omega)}}{\phi^{1/(\theta + \omega)}} = 0.15$$

Parameter	Value	origin	
α	0.6	Labor share.	
$\lambda$	1.16	TFP growth and patent hazard rate, $\phi$ .	
$g_l$	1.013	GDP growth.	
$\gamma$	0.015	Share of R&D GDP.	
$\bar{r}$	0.04	$\beta = 0.96$ from the literature.	
ρ	0.92	Average time until citing a scientific paper.	
$\rho_z$	0.5	Persistence of TFP shocks $(AR(1))$ .	
$\sigma_{\epsilon}$	0.6	Variance of TFP shocks $(AR(1))$ .	
ω	0.57	Ratio Basic/Applied and R&D Wage/Average Wage.	
$\kappa$	0.9	Arbitrary choice.	
δ	0.3	Arbitrary choice.	
θ	0.8	Ratio Basic/Applied and R&D Wage/Average Wage.	
η	0.15	Patenting (hazard) rate on the BGP.	

Table 4: [Calibratino results - parameter values]Parameter Values

# 5.3 Calibration Results

I apply a second-order perturbation to the model using Dynare. The following graphs show the dynamic behavior of key variables after a positive shock (high realization of Z):

Figure 3: Simulation Results - Response to a Positive Shock

Now, we can use the model to study the impact of the shock on TFP dynamics. For that purpose, we can rewrite the output as follows:

$$Y = exp(z)exp(\int_0^1 ln(y_i)di) = \underbrace{e^z e^{\bar{Q}}}_{TFP} L^{\alpha} K^{1-\alpha}, \quad \bar{Q} = \int_0^1 ln(q_i^*)di$$

Using that, we can decompose GDP growth:

$$g_y = \frac{Y'}{Y} = \frac{e^{z'}}{e^z} g_l \lambda^F \approx \underbrace{e^{Z'-Z}}_{\substack{\text{TFP} \\ \text{shock}}} + \underbrace{g_l}_{\substack{\text{Labor} \\ \text{frowth}}} + \underbrace{\lambda^F}_{\substack{\text{Innovation}}}$$

The innovation component can be further broken down into the following:

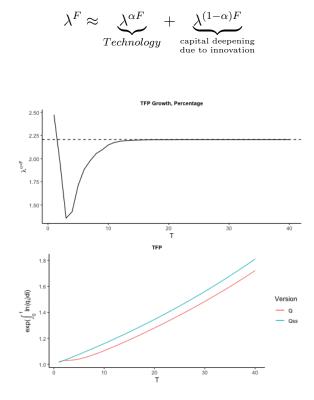


Figure 4: Simulation Results - TFP Respons to a Positive Shock

In this scenario, when a shock impacts the economy, there is a temporary surge in the technology growth rate, increasing by approximately 10% - from a steady-state growth rate of 2.25 to 2.5. However, this rise is followed by a downturn that persists for several periods. The original TFP (total factor productivity) growth rate is regained after a decade. Still, the TFP level never fully recovered to what it would have been without the positive shock.

# 6 Conclusion

At the core of this paper is the understanding that if growth indeed results from economic activity, as many believe, separating its analysis from transient fluctuations (cycles) is unwarranted. Instead of disregarding the alleged irrelevance of cycles to long-term performance, we should aim to reconcile it with existing theories by rethinking the nature of R&D. In this paper, I have done so by focusing on two features of R&D activity. First, I documented that R&D composition changes in response to the cycle and that some types of privately funded R&D (basic research) are countercyclical. If, as I propose, long-term R&D is more likely to produce substantial technological advances, an improvement in the allocation of R&D towards activities with greater social value counters the decline in R&D investment during a recession. These forces can neutralize each other. Second, I explicitly incorporated into the model a mechanism through which the productivity of R&D activity declines over time. This, too, emerges as a potential explanation for why shocks seem to have no lasting impact.

The next step in this research is to test it using firm-level data. A continuation project is underway using microdata at the US Census firm level. I believe employing such data will allow for better discernment of the various forces impacting R&D activity during the cycle. Furthermore, utilizing cross-sectional heterogeneity between sectors may improve our understanding of which fundamental features of production technology determine how R&D composition responds to the cycle and how that, in turn, shapes the future growth trajectory.

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# A Appendix

### A.1 Solving the Downstream Firm Problem

Since this is a Cobb-Douglas production function:

$$K = L \cdot \frac{(1-\alpha)}{\alpha} \cdot \frac{W}{r}$$

Also, due to constant returns to scale, the marginal cost of production is constant. We can generate it from the cost of producing a single unit, which will be given by (plugging in our expression for K):

$$1 = (Q \cdot L)^{\alpha} \cdot \left(L \cdot \frac{(1-\alpha)}{\alpha} \cdot \frac{W}{r}\right)^{1-\alpha}$$

Thus, we can solve for L

$$L = \frac{\left(\frac{1-\alpha}{\alpha}\frac{w}{r}\right)^{\alpha-1}}{q^{\alpha}}$$

And for K

$$K = \frac{\left(\frac{1-\alpha}{\alpha}\frac{w}{r}\right)^{\alpha}}{q^{\alpha}}$$

Using both, we find that the marginal cost of production is given by:

$$rK + wL = r \frac{\left(\frac{1-\alpha}{\alpha} \frac{w}{r}\right)^{\alpha}}{q^{\alpha}} + W \frac{\left(\frac{1-\alpha}{\alpha} \frac{w}{r}\right)^{\alpha-1}}{q^{\alpha}}$$

## A.2 Additions to the proof of theorem 4.1

The household Lagrangian is given by:

$$L = \frac{(c^{\alpha}l^{1-\alpha})^{1-\sigma} - 1}{1-\sigma} + \beta \mathbb{E}V(\vec{S}', a') + \mu(c + \frac{a'}{1+\bar{r}} - W_i(\vec{S})(1-l) - a)$$

Taking FOCs:

$$\partial L/\partial c: \gamma c^{\gamma(1-\sigma)-1} l^{(1-\gamma)(1-\sigma)} = \mu P(\overrightarrow{S})$$

$$\partial L/\partial l: (1-\gamma) l^{-\gamma-\sigma(1-\gamma)} c^{\gamma(1-\sigma)} = \mu W_i(\overrightarrow{S})$$

$$\partial L/\partial a': \beta \mathbb{E} \frac{\partial V(\overrightarrow{S}',a')}{\partial a'} = \frac{\mu}{1+\bar{r}}$$

Using ENT:

$$\frac{\partial V(\overrightarrow{S},a)}{\partial a}=\mu$$

Combining the first two conditions, we get the intra-temporal optimality condition:

$$l = \frac{1 - \gamma}{\gamma} \frac{P(\overrightarrow{S})c}{W_i(\overrightarrow{S})}$$

Combining the ENT with the FOC, we get the Euler.

$$\frac{c^{\gamma(1-\sigma)-1}l^{(1-\gamma)(1-\sigma)}}{P(\overrightarrow{S})} = (1+\overline{r})\beta \mathbb{E}(\frac{(c')^{\gamma(1-\sigma)-1}(l')^{(1-\gamma)(1-\sigma)}}{P(\overrightarrow{S}')})$$

## A.3 Further Analysis of the Standardized Model

Let:

$$C1 = \left(\frac{\left(\frac{1+r}{\lambda^F} - \rho\right)}{\delta\kappa\left(\frac{g_l - \rho}{\delta}\right)^{(\kappa-1)/\kappa\theta}} - \frac{(1-\kappa)\left(\frac{g_l - \rho}{\delta}\right)^{1/\kappa}}{\kappa\theta}\right)\omega$$
$$C2 = \left(\frac{g_l - \rho}{\delta}\right)^{1/\kappa}$$

And define  $\phi$ , the SS value of F, as the solution to the equation:

$$\phi - \left(\frac{\left(\frac{1+r}{\lambda^{F}} - \rho\right)}{\delta\kappa(\frac{g_{l}-\rho}{\delta})^{(\kappa-1)/\kappa}\theta} - \frac{(1-\kappa)\left(\frac{g_{l}-\rho}{\delta}\right)^{1/\kappa}}{\kappa\theta}\right)\omega^{-\theta}\left(\frac{g_{l}-1}{\eta}\right)^{\theta+\omega} = 0$$
(35)

We can follow similar steps to those shown above and find a steady state. Specifically, given that the conditions in 4.1 are satisfied, and in addition:

C5. Discounting on the BGP (Is this condition necessary?):

$$\frac{g_l \lambda^F}{1+\bar{r}} < 1$$

the steady state is given by the following:

The SS is defined by the following set of equations:

$$F^* = \phi \tag{36}$$

$$n* = \frac{\gamma}{C1 + C2} \tag{37}$$

$$r_a^* = \frac{C1\gamma}{C1+C2} \tag{38}$$

$$r_b^* = \frac{C_2 \gamma}{C1 + C2} \tag{39}$$

$$U^* = \frac{\eta r_a}{g_l - 1} = \frac{\eta \gamma C 1}{(C1 + C2)(g_l - 1)}$$
(40)

$$\pi^* = (1 - \lambda^{-1})e \tag{41}$$

$$\nu^* = \frac{(1 - \lambda^{-1})e}{1 - \frac{g_l \lambda^{\phi}(1 - \phi)}{1 + \bar{r}}} \tag{42}$$

 $Y = e \tag{43}$