The Impact of Business-Cycles on R&D Composition and Growth Through Technical Change

November 9, 2022

Motivation

R&D expenditure is cyclical; basic research expenditure is counter-cyclical:

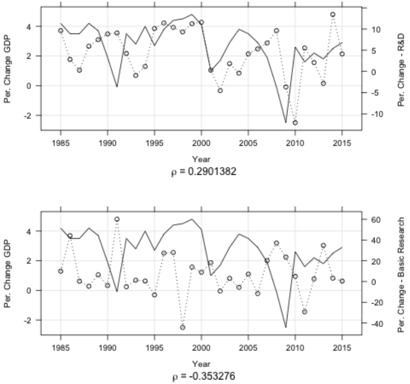


Figure: Source: BRDIS/SIRD (US Census); Private Firms Research Expenditure.

Research Questions:

1. Why is basic research counter-cyclical?
2. What is the impact of business-cycle fluctuations on future growth through technical change?

Data:

- Census firm-level data: BRDIS, LBD, QFR.
- Patent and scientific publications data.

Analysis:

- Reduced-form: shock → R&D composition → Innovation.
- Calibrate an endogenous growth model with different types of R&D.

Modelling R&D

- Declining productivity over time.

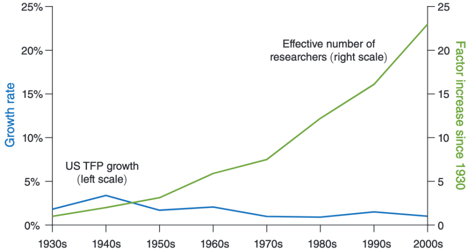


Figure 1. Aggregate Data on Growth and Research Effort

- Substantial heterogeneity in the nature R&D.
- Basic research - knowledge accumulation; Applied research - creating new goods and modes of production.

Related Literature

1. Long-term impact of short-term fluctuations via technology: Barlevy (2002), Comin-Gertler (2006), Queralto (2018, 2019).
2. Endogenous growth: Ackigit et. al. (2016), Kortum (1997), Romer(1990).
3. R&D throughout the business cycle: Aghion et. al. (2012), Manso et. al. (2017), Rafferty (2003), Schumpeter (1939).

Data

SVAR

$$y_t = A_1 y_{t-1} + A_2 y_{t-2} ... A_j y_{t-j} + B\epsilon_t$$

Where $y_t = [GDP, R\&D]'$ (Per. change), ϵ - orthogonal shocks.

Identifying assumption: $B_{1,2} = 0$.

Estimates for $B_{2,1}$:

R&D Variable	1956-2015	1985-2015
Basic Research	-0.024 (0.02)	-0.08 (0.037)**
Ratio Basic/Applied Research	-0.053 (0.023)**	-0.0124 (0.033)***
All R&D	0.0137 (0.0056)**	0.0009 (0.008)

Model

Simplified Version of the Model

1. An economy with an infinite number of periods.
2. L households. Provide manual and R&D labor. Grow exogenously at a rate of g_l
3. Output: $Y = e^Z QL$;

   L - labor, Q - technology, Z - shock.
4. $Q' = Q\lambda^F$;

   F - Blueprints. Produced by firms using R&D labor.
5. Blueprints are leased for $(1 - \lambda^{-1})Y$.

   The right to lease is lost with a probability of F in each period.

Technology production function

n - knowledge. U - Knowledge utilization.

r_a - applied research. r_b - basic research.

Blueprints production function:

$$F = r_a^{\omega} \frac{n^{\theta}}{U^{\omega+\theta}}$$

$$s.t: \quad n' = \rho n + \delta r_b^{\kappa} n^{1-\kappa} + \delta_e R_b$$

Also:

$$U' = U + \eta r_a$$

BGP

1. **Declining productivity of R&D**:

   On the BGP, r_a, r_b, n, U grow at a rate of g_l.

   $$F = r_a^{\omega} \frac{n^{\theta}}{U^{\omega+\theta}}$$

   remains constant

2. **Endogenously increasing R&D expenditure**:

   Higher returns from blueprints due to the expansion of the economy.

Model Dynamics

1. Low Z $\rightarrow$ Lower returns from blueprints .
   - less r_a $\rightarrow$ slower growth of Q and U.
   - lower opportunity cost of r_b $\rightarrow$ More r_b $\rightarrow$ Higher n.
2. Higher n + lower U $\rightarrow$ Higher productivity of R&D in successive periods.
3. The higher productivity compensates for some of the decline in innovation during the downturn.
4. The long-term impact of a downturn is ambiguous: lowers R&D, but mitigates inefficiency in composition.

The Economy

1. Infinite number of periods, $t = 0, 1, 2, ...$
2. Continuum of size 1 of households. Provide labor and consume the final good. Effective labor units exogenously increase at a rate of g_l.
3. Continuum of size 1 of intermediate goods.
4. Three types of firms:
   - continuum of R&D firms - Use R&D labor to create new technologies for producing the intermediate goods.
   - Continuum of upstream firms - Buy the new technologies and produce intermediate goods using capital and manual labor.
   - A single downstream firm - Combines the intermediate goods to a final good. Subject to exogenous production shocks, Z.

Households

1. CRRA utility function:

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\frac{C_t^{1-\sigma}-1}{1-\sigma}$$

   Choose how much to consume c, and how much to save/borrow, a.

2. Own the capital and the firms.

3. Endowed with one unit of manual labor, and γ units of R&D labor.

   No disutility from labor.

Downstream Producer

1. CRS production function:

$$Y = Zexp(\int_0^1 ln(y_i)di)$$

y_i - quantity of intermediate good i.

2. Cobb-Douglas production function, which implies:

$$y_i^d = \frac{E}{p_i^*} \quad (1)$$

Where E denotes expenditure on intermediate goods, and $p_i^* = min_{j \in J}\{p_{i,j}\}$.

3. Takes the price of the final good, P, as given.

Upstream Firms

1. An upstream firm, j, is an infinite vector of quality levels $\{q_{i,j}\}_{i\in[0,1]}$.
2. The production technology of firm j for good i:

$$y_i = (q_i l_i)^{\alpha}(k_i)^{1-\alpha}$$

3. Purchase patents - exclusive right of use in state-of-the-art technology $(q_{i,j*} > \max_{j\in J}\{q_{i,j}\})$; Patent are sold in a Bertrand competition.
4. If the state-of-the art technology improves further, the patent is infringed, and the second-best technology, denoted $\tilde{q}_i$, becomes available to all.
5. Assume: $\frac{q_i^*}{\tilde{q}_i} = \bar{\lambda}$

Patent Holder Problem

- The patent holder maximizes:

$$\pi(E, W_m, r, q_i^*, \tilde{q}_i) = \max_{\{p_i, y_i, l_i, k_i\}} \{(p_i y_i - W_m l_i - r k_i)\mathbb{I}\{p_i \leqslant min_{j \in J}\{p_{i,j}\}\}\}$$

$$\text{s.t :} \qquad y_i = \frac{E}{p_i}, \quad y_i = (q_i^* l_i)^\alpha k_i^{1-\alpha}$$

- Solving, we find:

$$\pi(E, W_m, r, q_i^*, \tilde{q}_i) = (1 - \bar{\lambda}^{-1})E$$

- Let $F := Pr(\text{patent infringed})$. The value of the patent:

$$\nu_t = \pi_t + \mathbb{E} \sum_{\tau=1}^{\infty} \frac{\prod_{t+1}^{t+\tau}(1 - F_{t+\tau})}{(1+r)^\tau} \pi_{t+\tau}$$

R&D firms

n - knowledge. U - Knowledge utilization.

r_a - applied research. r_b - basic research.

Probability of getting a patent:

$$F(U, n, r_a) = \min\left\{ r_a^{\omega} \frac{n^{\theta}}{U^{\omega+\theta}}, 1 \right\}, \quad \omega, \theta \in (0,1)$$

Knowledge accumulation:

$$n' = \rho n + \delta r_b^{\kappa} n^{1-\kappa} + \delta_o \int_{j\in J} r_{b,j} dj + \delta_a r_{b,a}^{\psi}, \quad \delta \in \mathbb{R}_+$$

Knowledge utiliztion:

$$U_{\tau} = \underline{U} + \sum_{t=0}^{\tau} \int_{j\in J} \eta r_{a,j,t} dj$$

.

R&D Firm's Problem

Let $\overrightarrow{S} = (Z, L, K, \bar{Q}, U, N)$, where $\bar{Q} = (\int_0^1 ln(q_i^{*\rho})di)$.

$$V(\overrightarrow{S}, n) = \max_{r_a, r_b}\{r_a^{\omega}\frac{n^{\theta}}{U^{\omega+\theta}}\nu(\overrightarrow{S}) - W_r(\overrightarrow{S})(r_a + r_b) + \frac{1}{1+r}\mathbb{E}[V(\overrightarrow{S}', n')]\}$$

$$\text{s.t} \qquad n' = \rho n + \delta r_b^{\kappa} n^{1-\kappa}, \qquad \overrightarrow{S}' = H(\overrightarrow{S})$$

Solving, we get the Euler equation:

$$\omega r_a^{\omega-1}\frac{n^{\theta}}{U^{\omega+\theta}}\nu(\overrightarrow{S}) =$$

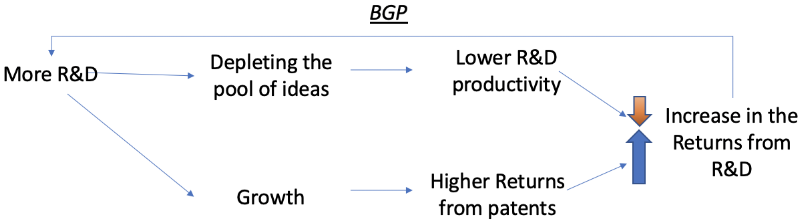
$$\delta\kappa(\frac{n}{r_b})^{1-\kappa}\mathbb{E}\Big[\frac{1}{1+r}r_a'^{\omega}\frac{n'^{\theta}}{U'^{\theta+\omega}}\nu(\overrightarrow{S'})\Big(\frac{\theta}{n'} + \frac{\rho\omega + \delta(1-\kappa)(\frac{r_b'}{n'})^{\kappa}}{r_a\delta\kappa(\frac{n'}{r_b'})^{1-\kappa}}\Big)\Big]$$

Equilibrium Definition

Existence of a BGP

Theorem: Existence and Uniqueness of a BGP

If the parameters of the model satisfy conditions $\mathbb{C}$ the economy has a unique BGP on which: r_a, r_b, n, U grow at a rate of g_l, $F = r_a^{\omega} \frac{n^{\theta}}{U^{\theta+\omega}}$ is constant, and Y, π grow at a rate of $g_l \lambda^F$, and $exp(\int_0^1 ln(q_i^*))$, grows as a rate of λ^F .



Stationary Model

Define $\tilde{x} = x/L$, normalize $Q = \int_0^1 ln(q_i)di = 1$ and $\tilde{S} = (Z, \tilde{L}, \tilde{K}, 1, \tilde{U}, \tilde{N})$.

Firm's problem:

$$V(\tilde{S}, \tilde{n}) = \max_{\tilde{r}_a, \tilde{r}_b} \{ \tilde{r}_a^{\omega} \frac{\tilde{n}^{\theta}}{\tilde{U}^{\omega+\theta}} \nu(\tilde{S}) - W_r(\tilde{S})(\tilde{r}_a + \tilde{r}_b) + \mathbb{E}[\frac{g_l \lambda^{F(\tilde{S}')}}{1+r} V(\tilde{S}', \tilde{n}')]\}$$

$$\text{s.t:} \quad g_l \tilde{n}_{t+1} = \rho \tilde{n}_t + \delta \tilde{r}_{b,t}^{\kappa} \tilde{n}_t^{1-\kappa}$$

$$\nu(\tilde{S}) = \pi(\tilde{S}) + \mathbb{E}[\frac{\lambda^{F(\tilde{S}'))} g_l}{1+r}(1 - F(\tilde{S}'))\pi(\tilde{S}') + ...], \quad F = \tilde{r}_a^{\omega} \frac{\tilde{n}^{\theta}}{\tilde{U}^{\omega+\theta}}$$

Also:

$$g_l \tilde{U}_{t+1} = \tilde{U}_t + \int_{j \in J} \eta \tilde{r}_{a,j,t} dj$$

$$\tilde{r}_a + \tilde{r}_b = 1$$

Calibration

Target Moments

Labor Share	0.6	BEA.
Patents' hazard rate	0.1	Kortum. (1997)
TFP growth	0.9%	BEA.
GDP growth	2.85%	BEA.
Share of R&D GDP.	0.015	BEA.
Real Interest Rate	1.04	Standard.
Average time until citing a scientific paper	9 years	Marx (2019).
Persistence of TFP shocks (AR(1))	0.5	BEA
Variance of TFP innovations (AR(1)).	0.6	BEA
Basic/applied research	0.23	SIRD (NSF).
Average R&D wage/average wage	3.12	SIRD (NSF).

Calibrated Parameters

α	0.6	Labor share.
λ	1.16	Step size.
g_l	1.013	Effective labor force growth.
γ	0.015	Share of labor force engaging in R&D.
$\bar{r}$	0.04	Interest rate.
ρ	0.92	Persistence of knowledge.
ρ_z	0.5	Persistence of TFP shocks.
σ_ϵ	0.6	Variance of TFP Innovations.
ω	0.27	Concavity of F in applied research
κ	0.9	Concavity of knowledge in basic research (arbitrary choice).

Calibrated Parameters - Continues

δ	0.3	Effectiveness of basic research (arbitrary choice).
θ	0.8	Concavity of F in knowledge.
η	0.15	Rate of knowledge utilization.

Growth Decomposition

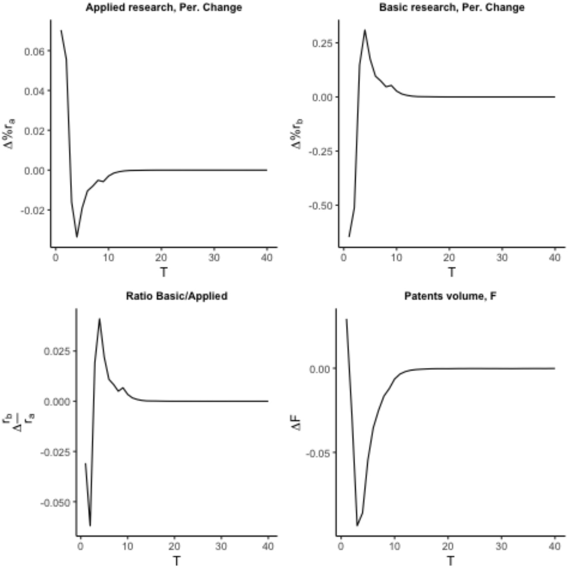
$$Y = Zexp(\int_0^1 ln(y_i)di) = \underbrace{Ze^{\bar{Q}}}_{TFP} L^{\rho} K^{1-\rho}, \quad \bar{Q} = \int_0^1 ln(q_i^*)di$$

Decomposing GDP growth:

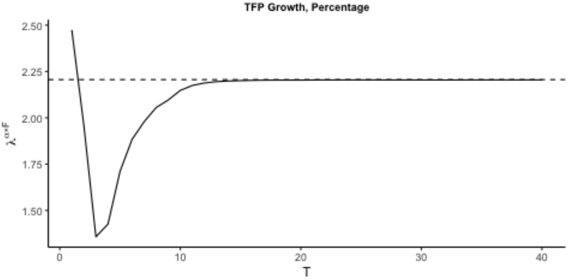
$$g_y = \frac{Y'}{Y} = \frac{Z'}{Z} g_l \lambda^F \approx \underbrace{\frac{Z'}{Z}}_{\substack{\text{TFP} \\ \text{shock}}} + \underbrace{g_l}_{\substack{\text{Labor} \\ \text{growth}}} + \underbrace{\lambda^F}_{\text{Innovation}}$$

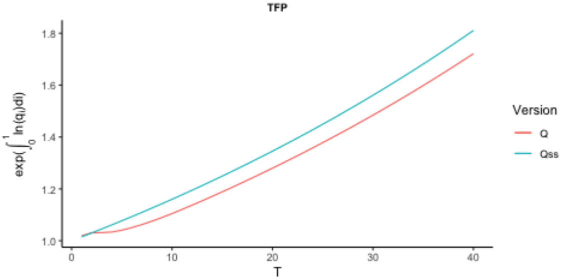
$$\lambda^F \approx \underbrace{\lambda^{\rho F}}_{Technology} + \underbrace{\lambda^{(1-\rho)F}}_{\substack{\text{capital deepening} \\ \text{due to innovation}}}$$

Simulation Results - IRF's



Simulation Results - TFP Growth





Next Steps

Possible Explanations and Their Identification

1. Transition to basic research to diminish costs.
   - Using cross-sectional variation in sales, credit, and R&D expenditure.
2. Lower profits due to depressed demands for output.
   - Correlation between sector-level output shocks and R&D composition.
   - Exploit differences in exposure to the domestic (vs. global) market.
3. Weaker demand from producers due to low availability of credit.
   - Negative correlation of basic research and credit; Increasing with the sector's dependence on external funding.
   - Weaker impact on large firms with good access to credit.
4. Uncertainty.
   - TBA

Appendix

Equilibrium Definition

A candidate for RCE will consist of:

1. A value function for the firm $V() : \mathbb{R}^7 \rightarrow \mathbb{R}$, and R&D policy functions $r_a(), r_b() : \mathbb{R}^7 \rightarrow \mathbb{R}_+$; Firms' average profit, $\Pi() : \mathbb{R}^6 \rightarrow \mathbb{R}_+$.
2. A pricing, production, labor demand and profit functions for a monopolist: $y_i(), p_i() : \mathbb{R}^7 \rightarrow \mathbb{R}$, $l_i(), \pi() : \mathbb{R}^6 \rightarrow \mathbb{R}_+$
3. Household value function: $V^h() : \mathbb{R}^7 \rightarrow \mathbb{R}$, and corresponding policy functions $A() : C() : \mathbb{R}^7 \rightarrow \mathbb{R}_+$.
4. Price of the final good, quantity produced, and total expenditure for the final good producer, $P(), Y(), E() : \mathbb{R}^6 \rightarrow \mathbb{R}_+$; A demand function for each intermediate good, $y_i^d() : \mathbb{R}^6 \times [0, 1] \rightarrow \mathbb{R}_+$.
5. A quality mapping: $Q() : \mathbb{R}^6 \times [0, 1] \rightarrow \mathbb{R}_+$, and a measure of goods

Equilibrium Definition

The candidate will consistute an equilibrium if the following holds:

1. $V(\overrightarrow{S}, k)$ solves the firm problem, with $r_a(\overrightarrow{S}, k), r_b(\overrightarrow{S}, k)$ being the corresponding policy functions; $\Pi(\overrightarrow{S})$ satisfies (5).
2. $V_h(\overrightarrow{S}, a)$ solves the HH problem, with $C(\overrightarrow{S}, a), A(\overrightarrow{S}, a)$ being the corresponding policy functions.
3. The monopolist is optimizing: $p_i(\overrightarrow{S}, q^*), y_i(\overrightarrow{S}, q^*)$ satisfy (2); $\pi(\overrightarrow{S})$ satisfies (3). $\rho W_m(\overrightarrow{S}) l_i(\overrightarrow{S}) = (1-\rho) r k_i(\overrightarrow{S})$.
4. The downstream producer is optimizing: $y_i^d(\overrightarrow{S}, q^*)$ satisfies (1); Makes a zero profit:

$$P(\overrightarrow{S}) * Y(\overrightarrow{S}) = E(\overrightarrow{S})$$

Defining Equilibrium- Feasability

5. The downstream producer expenditure is given by:

$$E(\overrightarrow{S}) = \int_0^1 y_i^d(\overrightarrow{S}, Q(\overrightarrow{S}, i))p_i(\overrightarrow{S}, Q(\overrightarrow{S}, i))di$$

6. Feasability of monopolist's choice:

$$(l_i(\overrightarrow{S})Q(\overrightarrow{S}, i))^\rho k_i(\overrightarrow{S})^{1-\rho} = y_i(\overrightarrow{S}, Q(\overrightarrow{S}, i))$$

7. Feasability of the downstream producer's choice:

$$Y(\overrightarrow{S}) = Zexp(\int_0^1 ln(y_i^d(\overrightarrow{S}, p_i(\overrightarrow{S}, Q(\overrightarrow{S}, i))di)$$

Defining Equilibrium- Consistency

8. For a measure $F(\overrightarrow{S})$ of goods:

$$Q(H(\overrightarrow{S}), i) = \bar{\lambda} Q(\overrightarrow{S}, i)$$

For the rest: $Q(H(\overrightarrow{S}), i) = Q(\overrightarrow{S}, i)$

9. The measure of new patents is given by:

$$F(\overrightarrow{S}) = r_a(\overrightarrow{S}, N)^{\omega} \frac{N^{\theta}}{U^{\omega+\theta}}$$

10. Consistency/representative agent:

$$H_1(\overrightarrow{S}) = U' = U + r_a(\overrightarrow{S})$$

$$H_2(\overrightarrow{S}) = N' = \rho K + r_b(\overrightarrow{S}, K)$$

$$H_3(\overrightarrow{S}) = K' = A(\overrightarrow{S}, A)$$

Defining Equilibrium- Market Clearing

11. Both labor markets clear:

$$r_a(\overrightarrow{S}, K) + r_b(\overrightarrow{S}, K) = \gamma L$$

$$\int_0^1 l_i(\overrightarrow{S}, Q(\overrightarrow{S}, i))di = L$$

12. The output market clears:

$$Y(\overrightarrow{S}) = C(\overrightarrow{S}, A) + A(\overrightarrow{S}, A) \qquad K'(\overrightarrow{S}) = A(\overrightarrow{S}, A)$$

13. The market for each intermediate good clears:

$$y_i^d(\overrightarrow{S}, p_i(\overrightarrow{S}, Q(\overrightarrow{S}, i))) = y_i(\overrightarrow{S}, Q(\overrightarrow{S}, i))$$

Back to the Firm's Problem

Existence of a BGP - Parametric Restrictions, $\mathbb{C}$

1. Consumption grows at a constant rate of λ^{ϕ}:

$$\beta(1+r^*)\frac{\lambda^{\phi(1-\sigma)}}{1+\phi(\lambda-1)}=1$$

2. The expected value of a patent is finite:

$$1+r^* > (1-\phi)(1+\phi(\lambda-1))g_I$$

3. R&D labor market clears:

$$\frac{1+r^*-(1-\phi)g_I(1+\phi)(\lambda-1)}{(1+r^*)\omega\phi} > \gamma$$

4. The probability of a patent is smaller than one:

$$0<\phi<1, \quad \phi := \left(\frac{\kappa\theta\delta^{1/\kappa}}{\omega(\frac{1+\bar{1}}{\lambda^{\phi}}-\rho)(g_I-\rho)^{\frac{1}{\kappa}-1}}\right)^{\theta}\left(\frac{g_I-1}{\eta}\right)^{\omega+\theta}$$

BGP, Uniqueness

The BGP is uniquely determined by the following conditions:

1. Euler:

$$r_a = \frac{\omega(\frac{1+\bar{1}}{\lambda^{F*}} - \rho)(g_I - \rho)^{\frac{1}{\kappa}-1}}{\kappa\theta\delta^{1/\kappa}} k$$

2. R&D labor market clearing:

$$r_a + r_b = \gamma L$$

3. Consistency:

$$gK = \rho K + \delta r_b$$

$$gU = U + \eta r_a$$

Characterizing the BGP

The BGP variables can all be written as a function of L:

$$C1 = \frac{\omega(\frac{1+\bar{1}}{\lambda^{F*}} - \rho)(g_I - \rho)^{\frac{1}{\kappa}-1}}{\kappa\theta\delta^{1/\kappa}}$$

$$C2 = \left(\frac{g_I - \rho}{\delta}\right)^{1/\kappa}$$

$K^*(L) = \frac{\gamma L}{C_1+C_2}$, $r_a^*(L) = \frac{C1\gamma L}{C1+C2}$, $r_b^*(L) = \frac{C2\gamma L}{C1+C2}$, $U^*(L) = \frac{C1\eta\gamma L}{(g_I-1)(C1+C2}$.

Further, we find that:

$$\frac{r_b^*}{r_a^*} = \frac{C_2}{C_1} = \frac{(\bar{g}_I - \rho)\theta\kappa}{\omega(\frac{1+r^*}{\lambda^F} - \rho)}$$

And:

$$\frac{W_r^*(L)(r_a^*(L) + r_b^*(L))}{Y^*(L)P^*(L)} = Cons.$$

The Process of R&D

1. Firms choose r_a, r_b.
2. Firms are randomly assigned a position in a queue.
3. Each firm in its turn targets a single good to innovate. It succeeds in doing do with probability $F(U, k, r_a)$. The probability is identical to all goods.
4. After seeing whether the firms succeeded or not, the next firm in line targets a specific good.
5. If the same good is improved by n firms, each gets the patent with probability $\frac{1}{n}$.

Back to R&D Firms.